

CARNEGIE
LEARNING



LONG + LIVE + MATH

**Reconsidering two sigma
EDM for the complete instructional system**

Steve Ritter
Founder and Chief Scientist

Carnegie Learning's History



- Founded @ Carnegie Mellon University 20 years ago
- Pioneers in the personalized and adaptive learning market
 - Intelligent Tutoring System
 - MATHia (was Cognitive Tutor)
- Over 500,000 students/year
- Not just math: CS, World Languages, ELA

Guiding Principle

The screenshot shows the MATHia software interface. On the left, a text box contains a word problem about a car stereo system. The problem asks for the initial cost and income, the cost and income for 100 stereos, the number of stereos sold for a given income, and the break-even point. Below the text are four numbered questions. In the center, a table shows the relationship between the number of stereos produced, the cost of production, and the income. The table has columns for 'Stereos Produced', 'Cost of Production', and 'Income'. The rows show the unit (stereos, dollars, dollars), the expression ($35s + 7350$, $210s$), and the values for Question 1 (0, 7350, 0), Question 2 (100, 10,850, 21,000), Question 3 (blank, blank, 23,100), and Question 4 (blank, blank, blank). On the right, a solver window shows the equation $210s = 23,100$ and the solution $s = 110$ after dividing both sides.

An electronics company has been designing a new car stereo system for several months. The design costs will be approximately \$7350. Each stereo will cost \$35 for material and labor. The company has decided to sell each stereo for \$210.

Define units for the cost of production and the income obtained from the stereos. Then enter a variable for the number of stereos produced and use this variable to write expressions for the cost of production and the income obtained from the stereos.

1. What is the initial cost of production? What is the initial income?
2. How much will the cost be and how much income would be obtained if 100 stereos are produced?
3. If the income was \$23,100, how many stereos were sold?
4. How many stereos must the company make and sell to break even?

After completing the worksheet, graph your model.

Quantity Name	Stereos Produced	Cost of Production	Income
Unit	stereos	dollars	dollars
Expression	s	$35s + 7350$	$210s$
Question 1	0	7350	0
Question 2	100	10,850	21,000
Question 3			23,100
Question 4			

Solve the equation for s .

Close Solution History

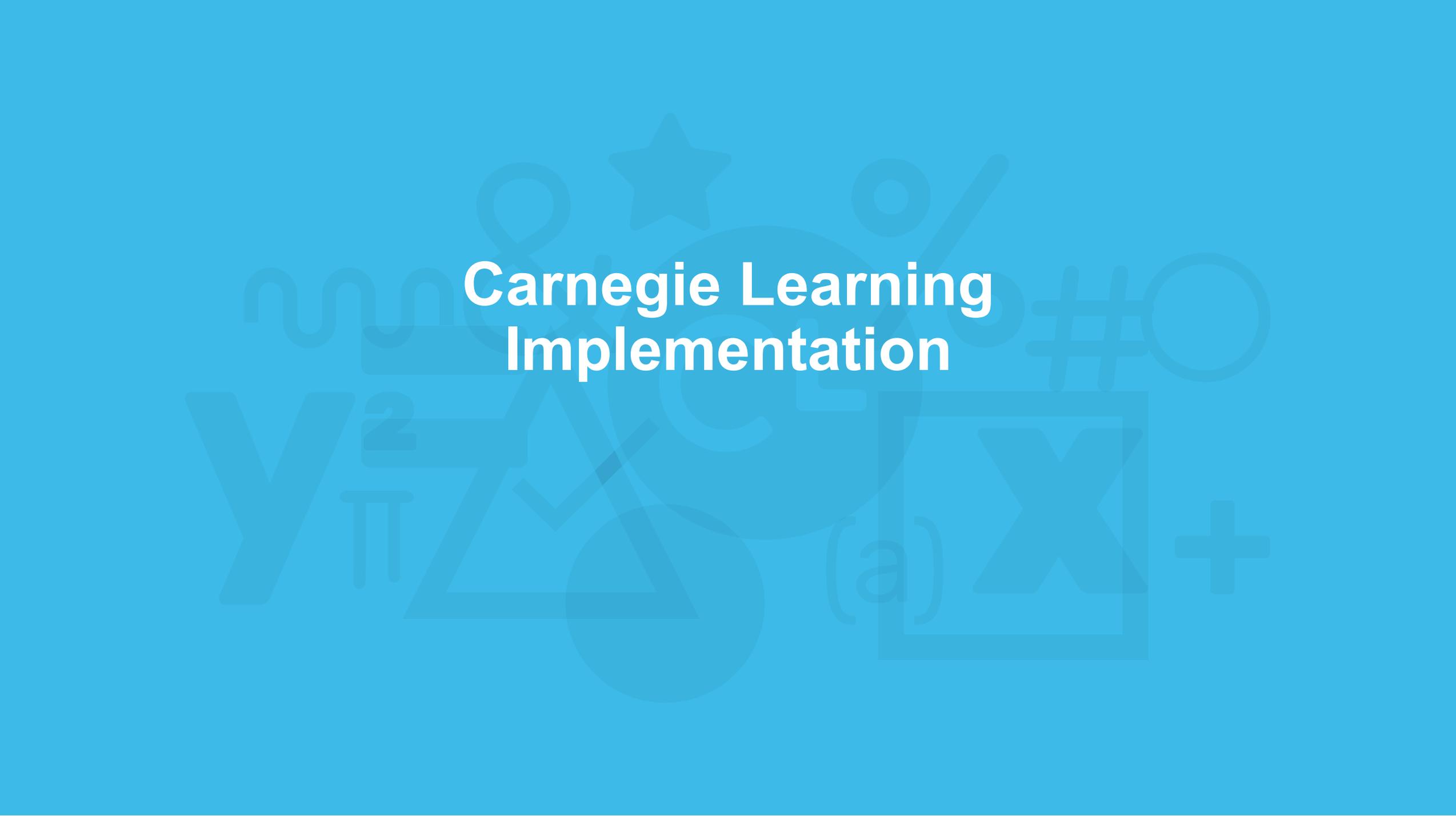
$$210s = 23,100$$

$$s = 110$$

Divide both sides

The more we understand about how students think and learn, the better we can help them think and learn

Carnegie Learning Implementation

The background is a solid blue color with various faint, light-blue mathematical symbols and icons scattered across it. These include a star, a circle with a percentage sign, a hash symbol, a large 'X' inside a square, a plus sign, a triangle, a circle, a wavy line, a large 'Y', a pi symbol, a square with a horizontal line, and a circle with a dot.

MATHia Approach

Active, Relevant and Adaptive

- Give students complex, real-world problems to solve
- Watch what students do as they solve them
- Provide feedback and opportunities to learn from errors

Mastery Learning

- Ensure students master prerequisites before moving on

The screenshot shows the MATHia software interface. The top navigation bar includes 'MATHia X', 'worksheet_grapher_x1_linear_systems_int', 'System Help', 'Glossary', and 'Curriculum Browser'. Below this is a secondary bar with 'Tour', 'Lesson', 'Step-by-Step', 'Solver', 'Hints', 'Solve It', 'Progress' (with a green progress bar), and 'I'm Done'.

The main content area is divided into three sections:

- Text:** A word problem about plumbers' costs and instructions to define units and write expressions.
- Table:** A table for defining variables with columns for 'Quantity Name', 'Job Length', '1st Plumber's Cost', and '2nd Plumber's Cost'. Rows are provided for 'Unit', 'Expression', and four 'Question' rows. Below the table are 'Draw Line' buttons.
- Graph:** A coordinate grid with a vertical axis labeled 'Dollars' and a horizontal axis. The vertical axis has 'Upper' and 'Lower' markers, and the horizontal axis has an 'Interval' marker.

At the bottom of the window, there is a footer with 'Problem: wgs71a009', 'Client Version: 1.11.03', 'Server Version: 1.11.03', and '© 2010 Carnegie Learning'.

MATHia Adaptation



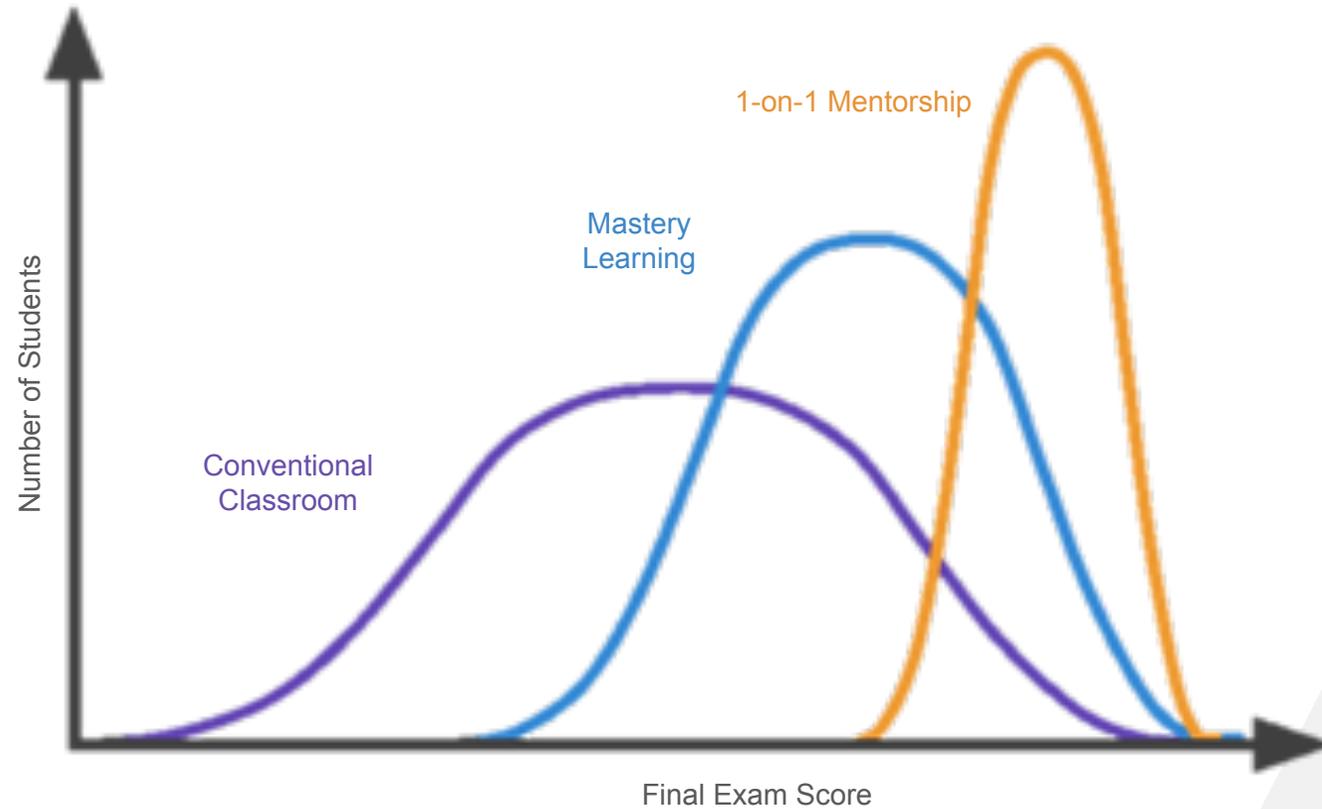
vanLehn, 2006

Blended Learning in Schools



Bloom's 2 Sigma Problem

the promise of intelligent tutoring systems



Bloom, 1984

Mastery based on knowledge components

MATHia[®] ratio_proportion_prop2 System Help Glossary Curriculum Browser

< Unit Overview Step-by-Step Sample Problem Hints Solve It Progress I'm Done

The Appalachian Trail is a 2,155-mile hiking trail in the Eastern United States. You plan to hike the section of the trail that is in New Jersey at a rate of 9 miles per day. If the hike will take you 8 days, what is the length of the trail in New Jersey?

Write a proportion to represent the problem.
Use a variable, such as y , to represent the unknown quantity.

<input type="text"/>	mile(s)	=	9	mile(s)
8	day(s)	=	<input type="text"/>	day(s)

Enter calculated quantity of rate.

Identify denominator of multiplicative identity.

Identify given denominator of rate in proportion.

Identify given denominator of unit rate in proportion.

Identify given numerator of rate in proportion.

Identify given numerator of unit rate in proportion. ✓

Identify given part in proportion.

Identify given total in proportion.

Identify label in denominator of proportion.

Identify label in numerator of proportion.

Strong results

CONTROL

.22

CARNEGIE LEARNING ALGEBRA I

.42

CARNEGIE LEARNING ALGEBRA I HIGHER COMPLETION*

.54

STANDARD DEVIATIONS

*Students completing more than 27 sections in Carnegie Learning Software

7 states

(AL, CT, KY, LA, MI, NJ, TX)

147 schools

**18,000+
students**

2 years

Data from second year

RAND/US Ed

Most students become very similar with regard to learning ability, rate of learning and motivation for learning – when provided with favorable learning conditions.

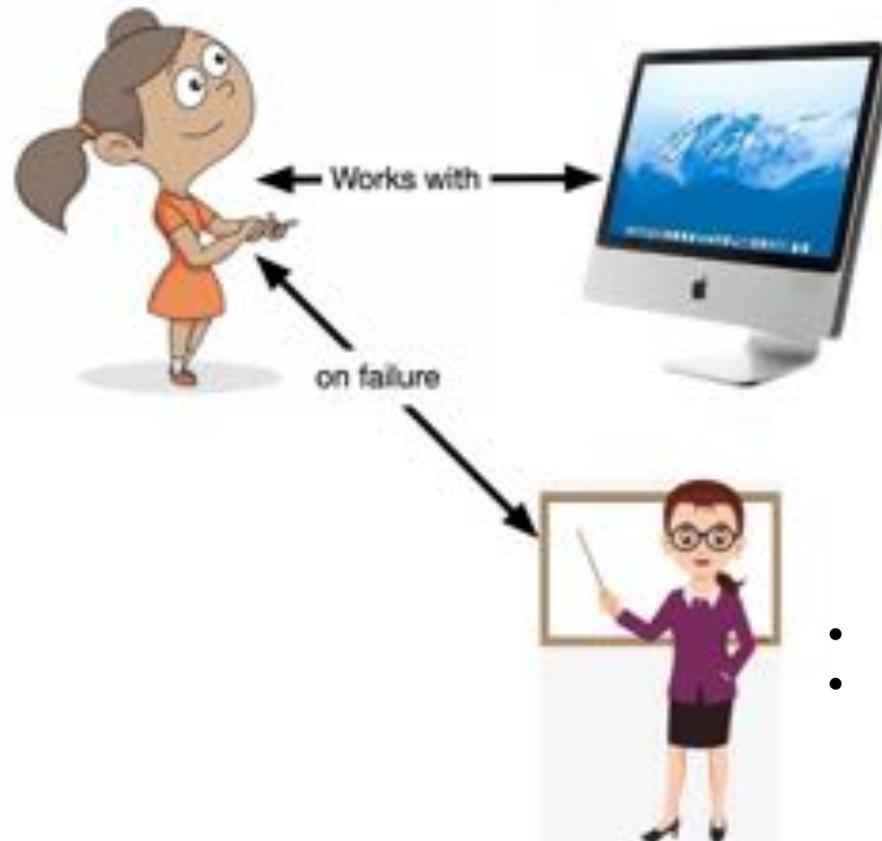
Bloom, 1976

Unfavorable learning conditions

- Frustration
- Boredom
- Lack of motivation for further learning
- Low confidence
- **Failure to master**
 - Student does not master all skills within some number of problems (25 or so)

Beck and Gong, 2013
Slater and Baker, 2019

Remediate on failure model



- Analytics: Teacher remediates
- Design: Software improves

Implementation Analytics

Unmastered Skill Summary

Module 1 | Module 2 | Module 3 | Module 4

Unit: [Unit] | Workspace: [Workspace] | Filter: Not Started

SKILLS	NOT STARTED	IN PROGRESS	REMEDATION SUGGESTED	NEAR PROFICIENT	PROFICIENT
Lorem ipsum dolor sit amet, consetetur sadipscing elitr >	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname
Smet, sadipscing elitr, sed pulsar elitera sekse	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname
Smet, sadipscing elitr, sed pulsar elitera sekse	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname	Lastname, Firstname
Smet, sadipscing elitr, sed pulsar elitera sekse				Lastname, Firstname	Lastname, Firstname
					Lastname, Firstname
					Lastname, Firstname

APLSE Report: All Students

DATE RANGE: From 08/15/16 To 10/02/16 | KEY: On-Track (Green), Approaching (Yellow), Off-Track (Red)

4 STUDENTS ON TRACK | 1 STUDENTS APPROACHING | 3 STUDENTS OFF TRACK

APLSE OVER TIME:

AVERAGE APLSE: 376 (as of 9/29/2016)
Out of 991
SCORE DISTRIBUTION: 50% On-Track, 12.5% Approaching, 37.5% Off-Track

AVERAGE WORKSPACES COMPLETED: 39

AVERAGE IMPROVEMENT: 376 (On-Track)

AVERAGE HINTS AND ERRORS: On-Track

AVERAGE PACE: On-Track

STUDENT DETAIL (as of 9/29/2016)

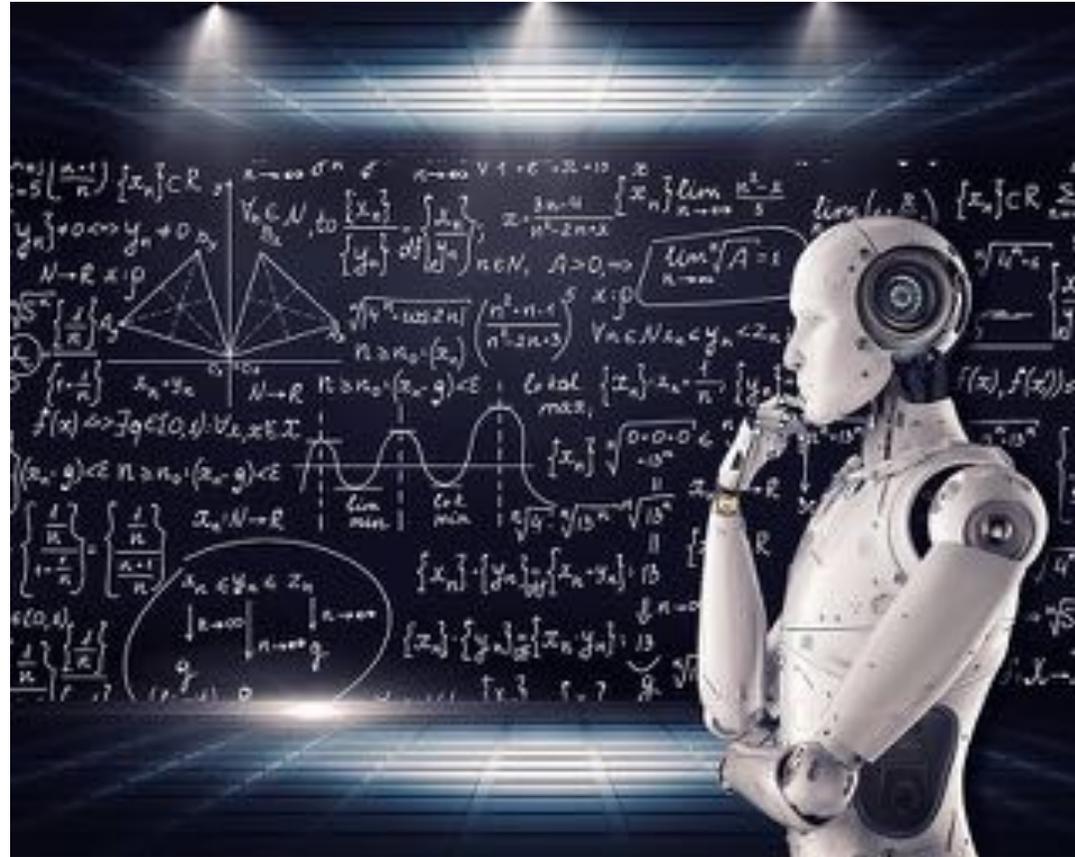
Student	Current APLSE	Improvement	Completed Workspa...	Pace	Hints + Errors
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Design adaptation



Aleven et al, 2017

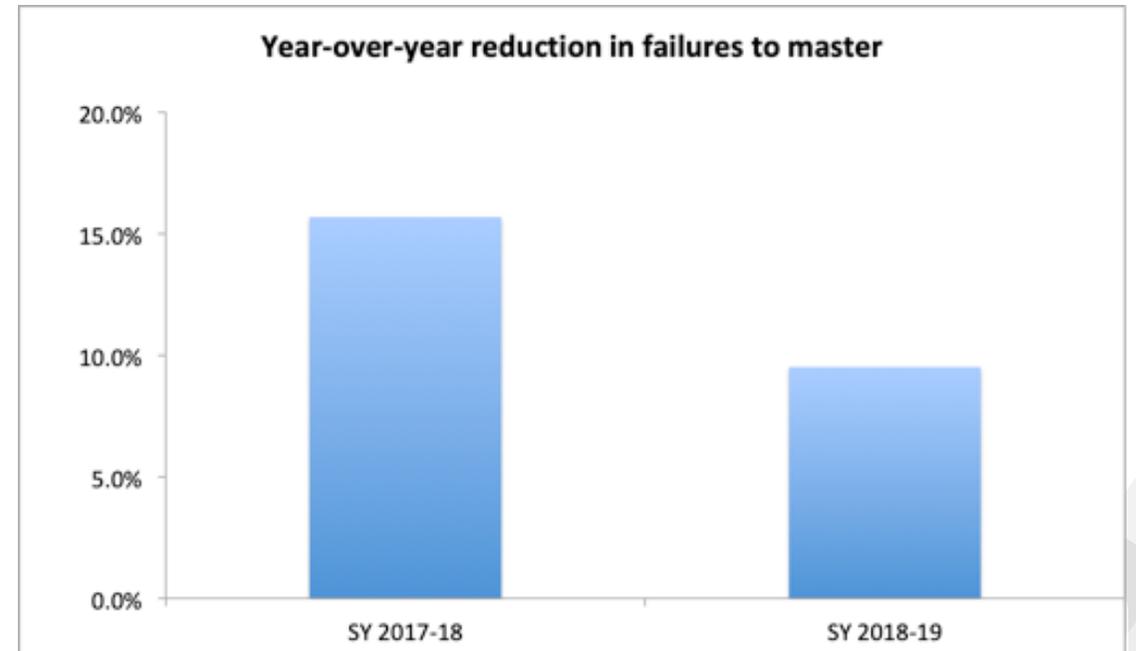
Strong AI



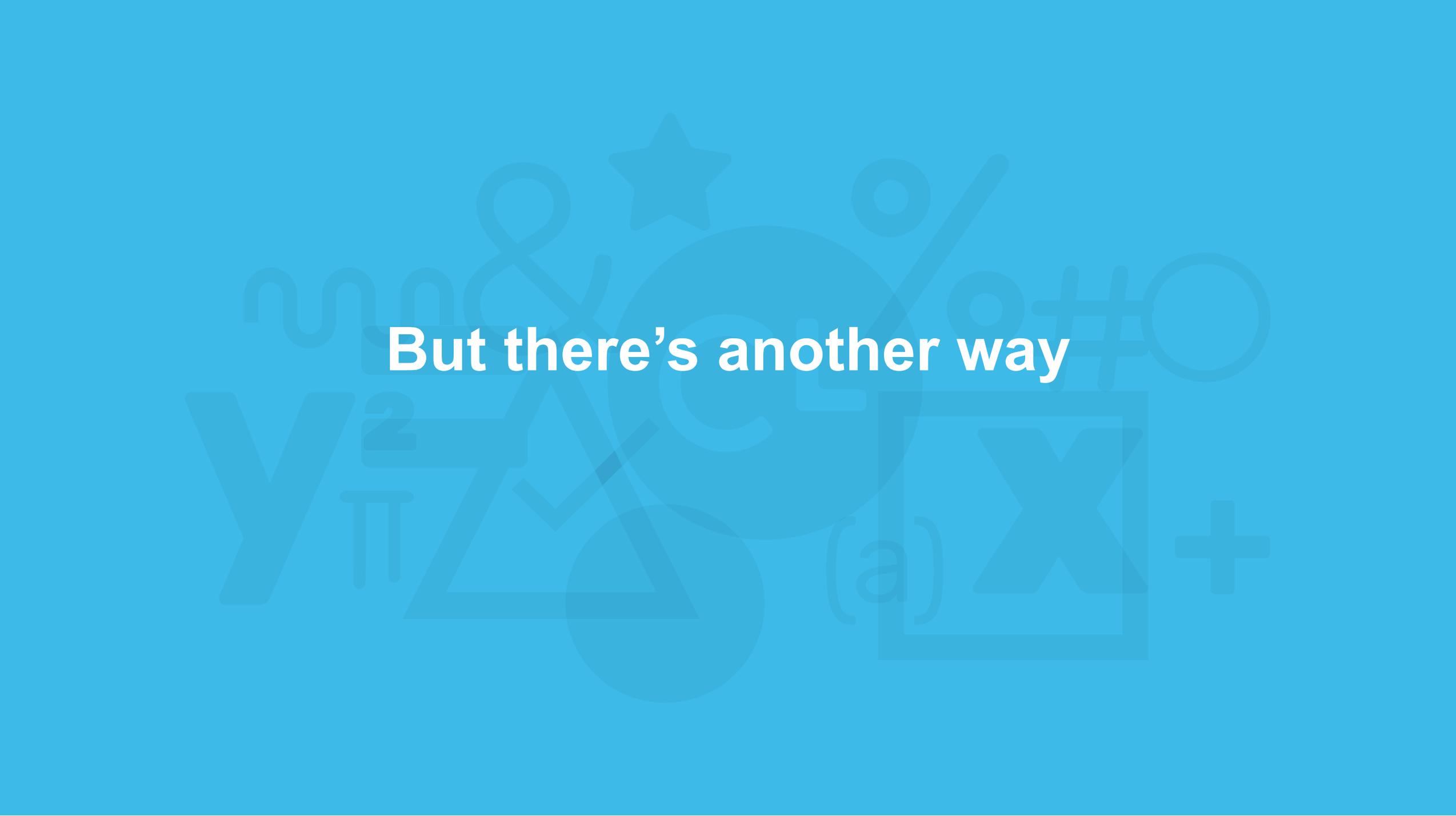
(a)
#R

Data-driven improvements

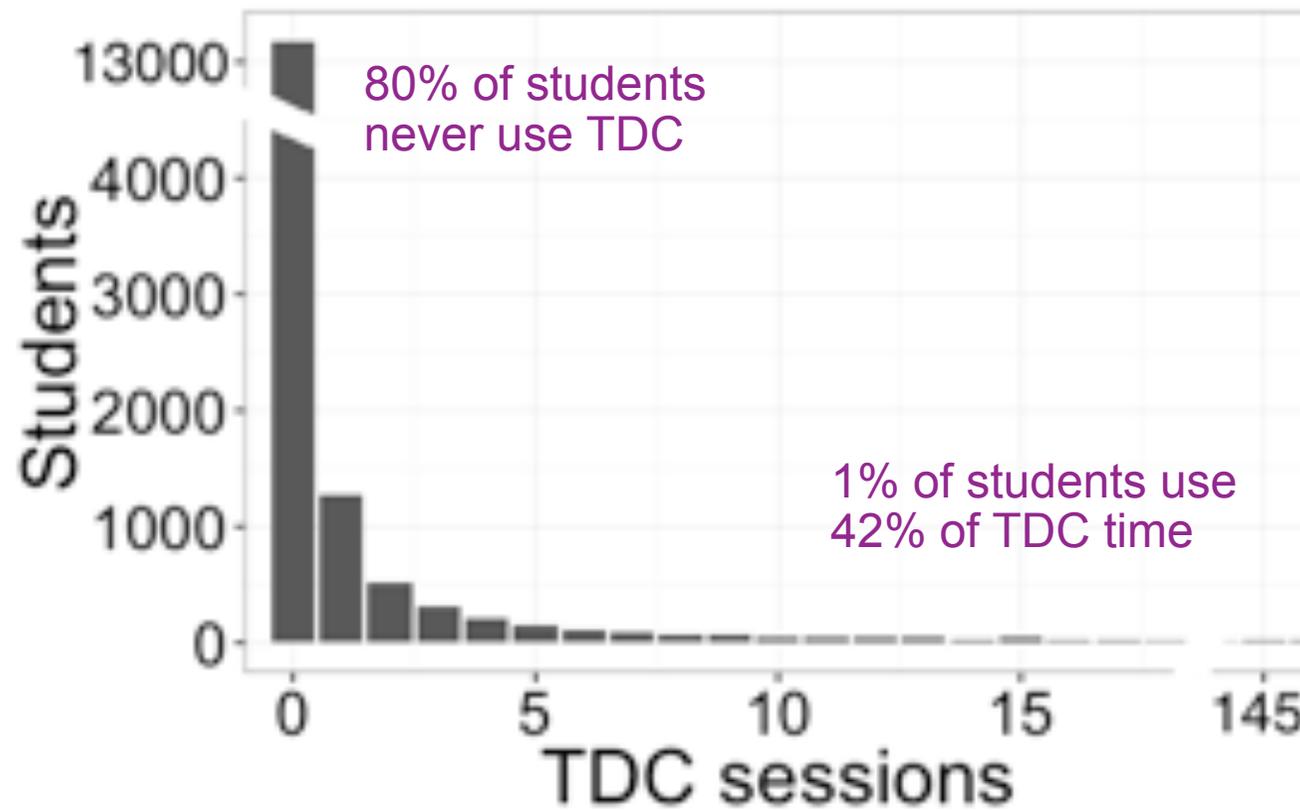
- 2019 Data
 - 4.4 million hours
 - 7.1 million workspaces
 - 63 million problems
 - 1.1 billion transactions



But there's another way

The background is a solid blue color with a collage of various mathematical symbols and shapes in a lighter blue, semi-transparent font. The symbols include an infinity symbol, a five-pointed star, a sine wave, a circle with a dot, a hash symbol, a large 'Y', a square with a horizontal line, a pi symbol, a triangle, a square, a plus sign, a circle with a dot, a large 'X' inside a square, and a plus sign.

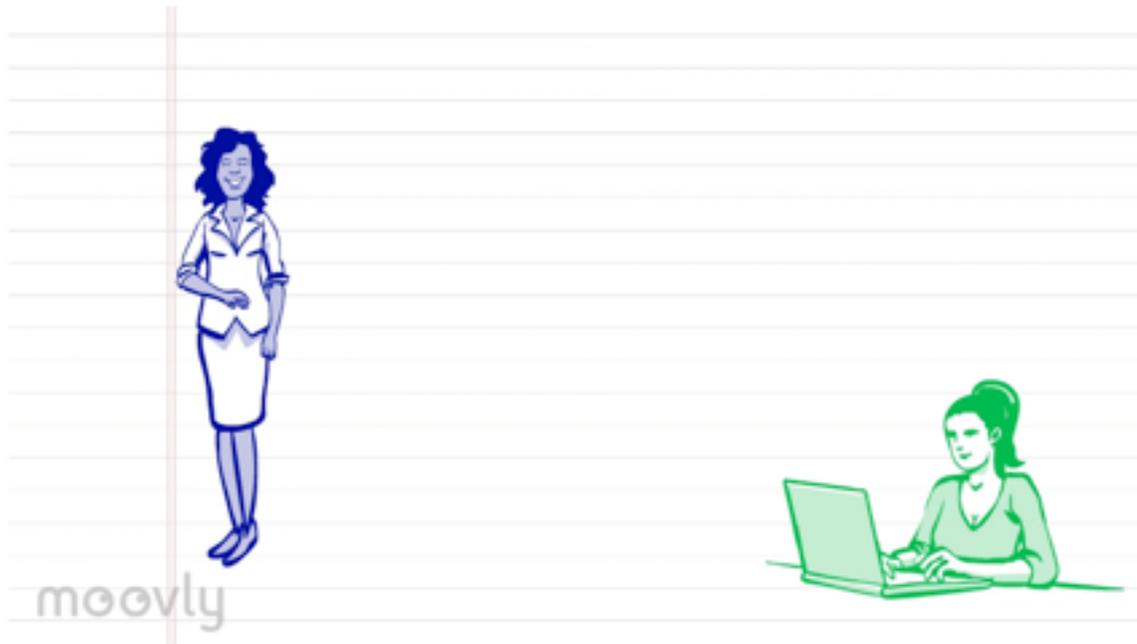
Teacher attention is poorly allocated



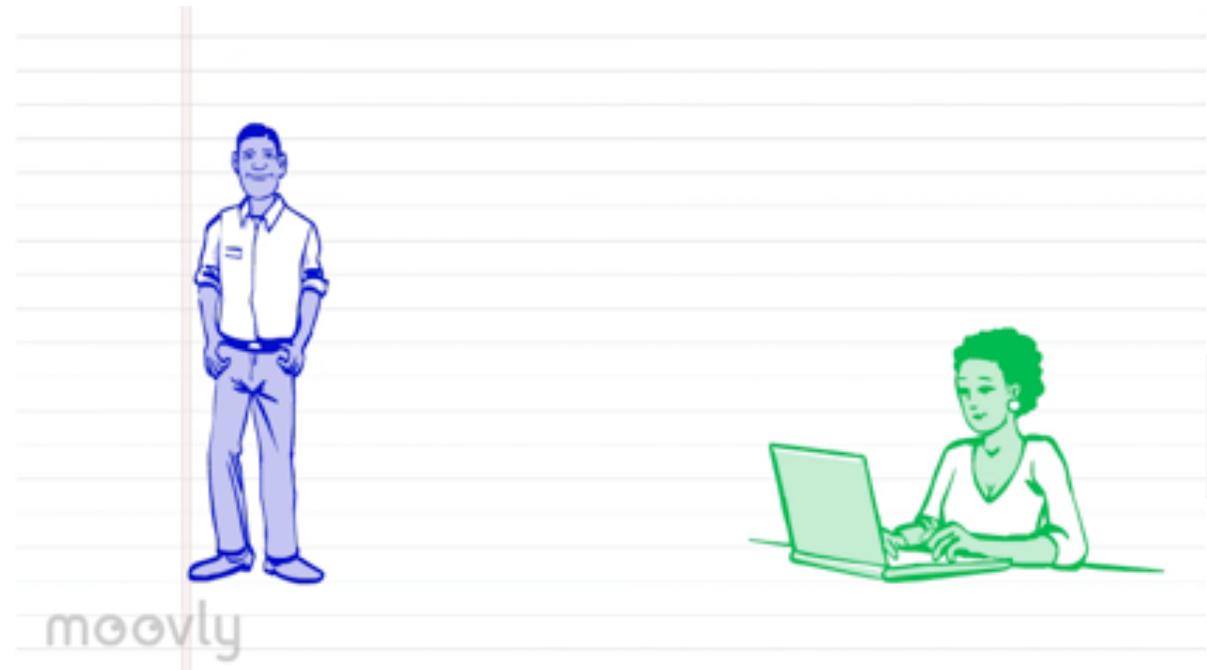
Fancsali, et al. EDM 2018

Information v. Affirmation

- [Information video](#)



- [Affirmation video](#)

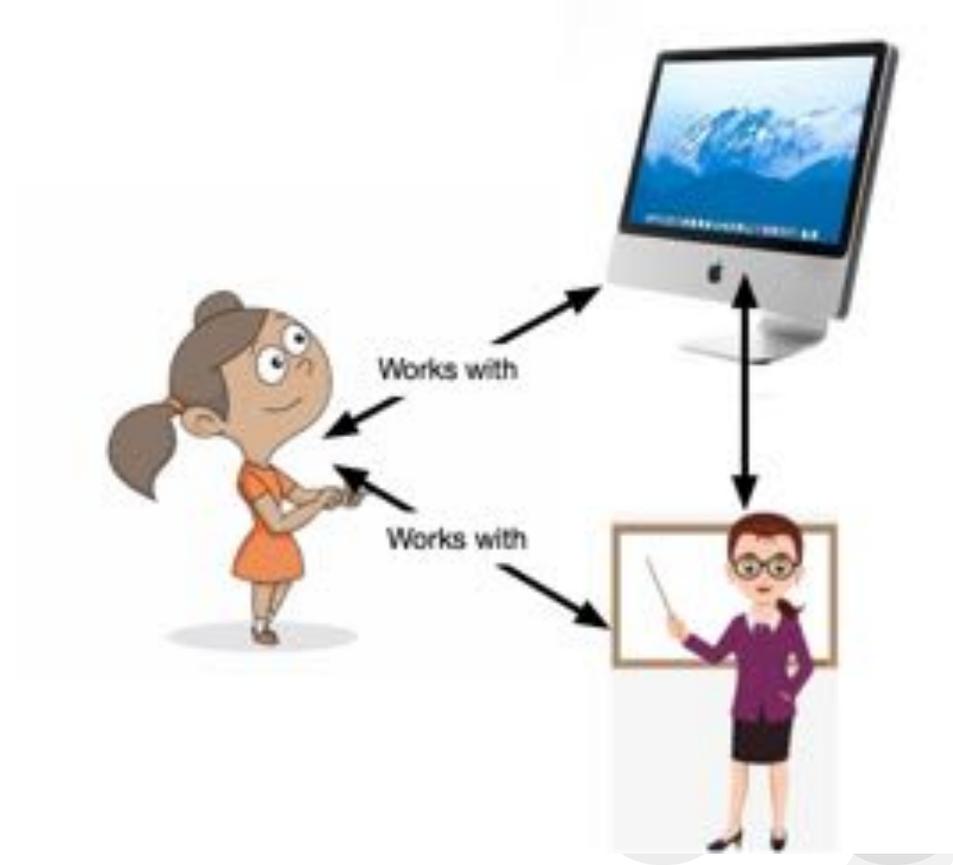


Information v. Affirmation

- **Tutor** Hi. How can I help you today?
 - **Student:** hello I am stuck on a problem I will load it so you can see
 - **Student:** I just need to know how to type the expression in the answer box
 - **Tutor:** Okay, you need plug 4.2 in $\frac{1}{2}h(6+4)$
 - 4.2 is height right?
 - **Student** yes i'm going to try it
 - **Student** 4.2 goes where
 - **Tutor** in h. since 4.2 is height
 - **Student:** would it been in ()
 - **Tutor** it will be 21
 - $\frac{1}{2}(4.2)(6+4) = 21$
 - **Student:** ok got cha
 - **Tutor:** so, that is it.
- **Tutor:** hi! what can I help you with today?
 - **Student:** Do you know how to do a factor table?
 - **Tutor:** Hmm I am familiar with it. Is there a problem that you wanted to go over, that you could write on the board for us?
 - **Student** This looks like an easy one, but I am not sure so I just want to make sure I understand this correctly
 - **Student:** To check this table is all you do multiply the top row by the 7x and see if it matches the bottom row? Is this right?
 - **Tutor:** Yeah everything looks good to me. Great job!
 - **Student:** I was hoping that I did this right.

Favorable learning conditions

- Acknowledges strengths of different instructional models
- Goals
 - Improve communication between teacher and software
 - Optimize teacher time



**Different use cases require
different analyses**

Design Adaptation Solution

The screenshot shows the MATHia X interface. On the left, a text problem is presented: "A local shop donated cell phone covers to the National Junior Honor Society to sell as a fundraiser. They sold each cover for \$3. At the end of the selling period, you also donated \$3 in cash. Define a unit for the amount raised. Then enter a variable for the number of cell phone covers sold and use this variable to write an expression for the amount raised." Below the text are three questions: "1. How much money did they raise if they sold 31 cell phone covers?", "2. How much money did they raise if they sold 23 cell phone covers?", and "3. How many cell phone covers did they sell if they raised \$114?". On the right, a table shows the data for these questions. The 'Expression' row for the amount raised is circled in red, showing the expression $3x + 3$. At the bottom right, a 'Skills' window is open, showing progress bars for various skills. The skill 'Enter given.' is highlighted in green with a checkmark, indicating it is mastered.

Problem: wgs1p2ae212 Client Version: 1.11.63 Server Version: 1.11.63

Improve knowledge component model

Learning curve

The screenshot shows the MATHia X interface. On the left, there is a text problem about cell phone covers. On the right, there is a table with columns 'Quantity Name', 'Cell Phone Covers Sold', and 'Amount Raised'. The 'Amount Raised' column contains the values 96, 72, 114, and the expression $3x + 3$, which is circled in red. Below the table are three questions related to the problem.

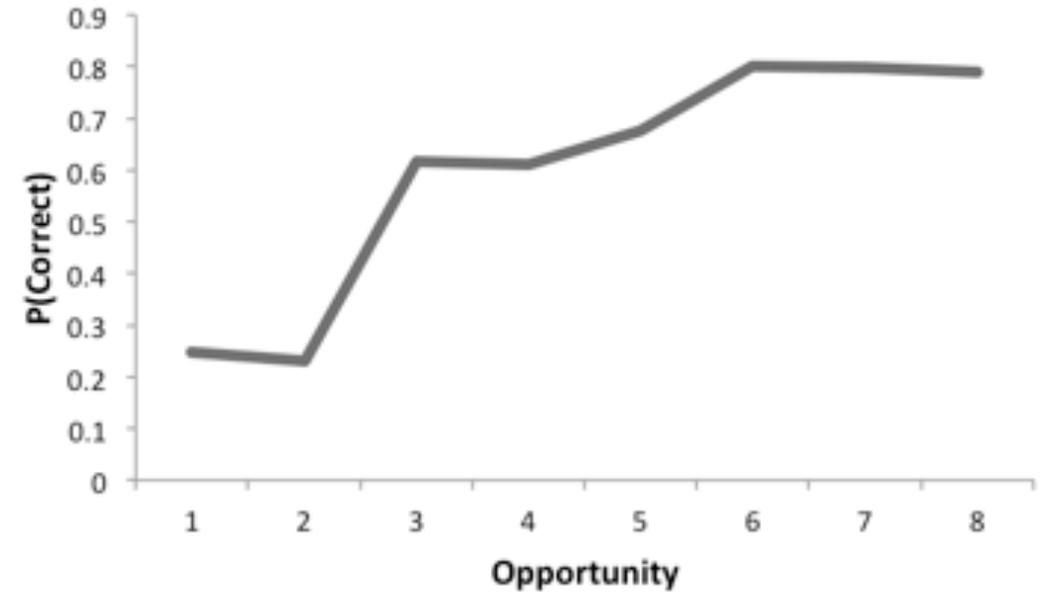
Quantity Name

	Cell Phone Covers Sold	Amount Raised
Unit	covers	dollars
Question 1	31	96
Question 2	23	72
Question 3	37	114
Expression	x	$3x + 3$

1. How much money did they raise if they sold 31 cell phone covers?

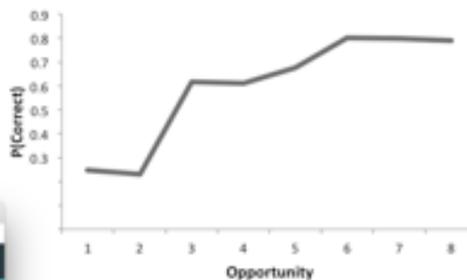
2. How much money did they raise if they sold 23 cell phone covers?

3. How many cell phone covers did they sell if they raised \$114?



Cen, Koedinger, Junker, 2006

Splitting knowledge components



MATHia X worksheet_grapher_at_patterns_3step_exp

System Help Glossary Curriculum Browser

Tour Lesson Step-by-Step Solver Hints Solve It Progress I'm Done

A local shop donated cell phone covers to the National Junior Honor Society to sell as a fundraiser. They sold each cover for \$3. At the end of the selling period, you also donated \$3 in cash.

Define a unit for the amount raised. Then enter a variable for the number of cell phone covers sold and use this variable to write an expression for the amount raised.

Quantity Name	Cell Phone Covers Sold	Amount Raised
Unit	covers	dollars

1. How much money did they raise if they sold 31 cell phone covers?

2. How much money did they raise if they sold 23 cell phone covers?

3. How many cell phone covers did they sell if they raised \$114?

A line graph showing the probability of being correct, P(Correct), on the y-axis (ranging from 0 to 1) against the number of opportunities, Opportunity, on the x-axis (ranging from 1 to 8). The curve starts at approximately (1, 0.25), rises to (2, 0.7), (3, 0.8), (4, 0.85), (5, 0.9), (6, 0.85), (7, 0.9), and ends at (8, 0.88).

Opportunity	P(Correct)
1	0.25
2	0.7
3	0.8
4	0.85
5	0.9
6	0.85
7	0.9
8	0.88

MATHia X worksheet_grapher_at_patterns_3step_exp

System Help Glossary Curriculum Browser

Tour Lesson Step-by-Step Solver Hints Solve It Progress I'm Done

Shannon is an outdoor enthusiast. She is bird watching at a nature preserve. Shannon has \$24 and rents a pair of binoculars for \$4 per hour.

Quantity Name	Time Renting Binoculars	Money Left
Unit	hours	dollars
Question 1	1	20

Define units for the time that Shannon rents binoculars and the amount of money Shannon has left. Then enter a variable for the time Shannon rents binoculars and use this variable to write an expression for the amount of money Shannon has left.

1. If Shannon rents the binoculars one hour, how much money will she have left?

2. After 2 hours of bird watching, she spots her favorite bird, an oriole, and goes home. How much money does she have left?

A line graph showing the probability of being correct, P(Correct), on the y-axis (ranging from 0 to 0.9) against the number of opportunities, Opportunity, on the x-axis (ranging from 1 to 8). The curve starts at approximately (1, 0.22), rises to (2, 0.55), (3, 0.7), (4, 0.78), (5, 0.8), (6, 0.82), (7, 0.85), and ends at (8, 0.82).

Opportunity	P(Correct)
1	0.22
2	0.55
3	0.7
4	0.78
5	0.8
6	0.82
7	0.85
8	0.82

Putting the teacher in the loop

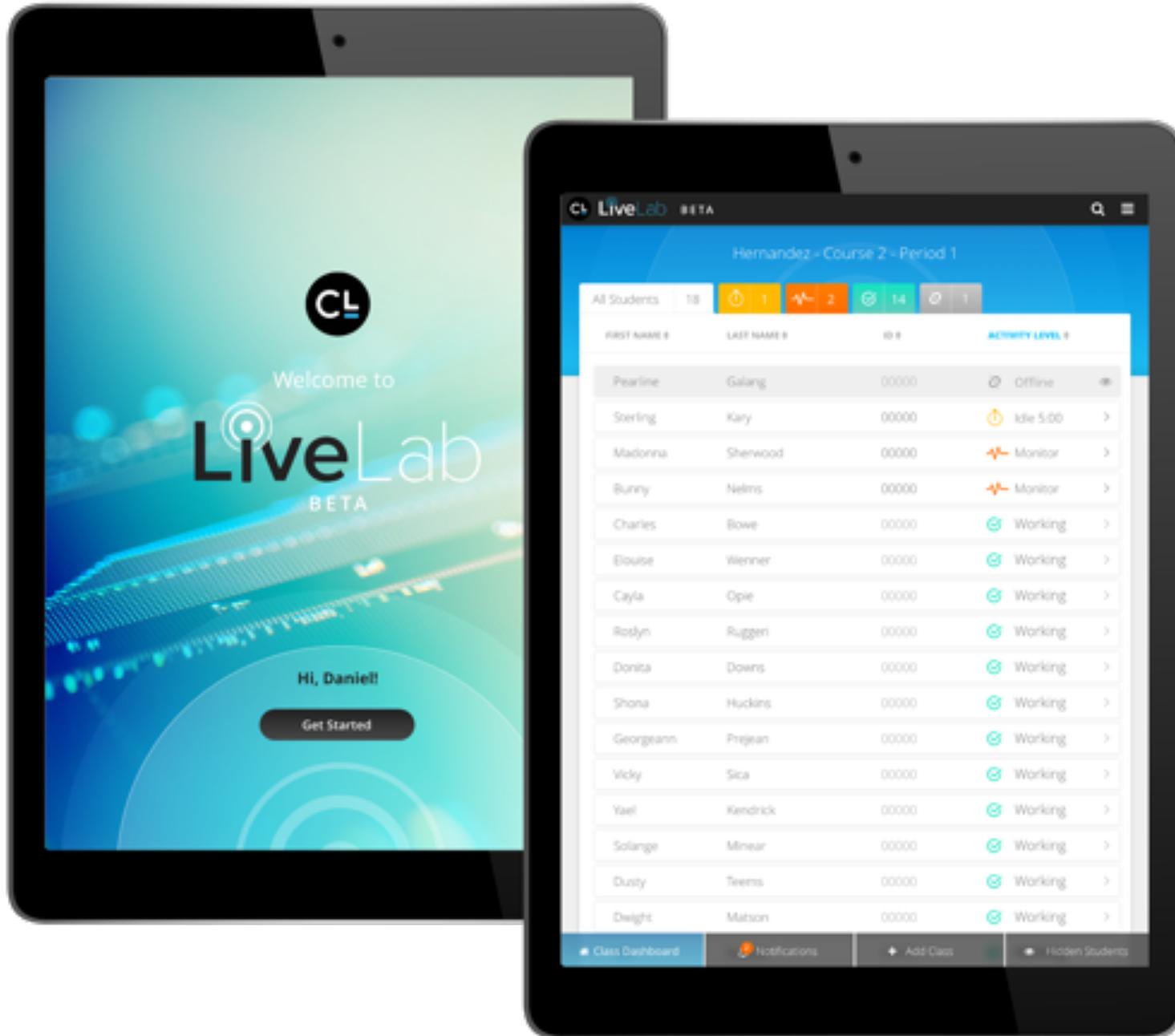


Dillenbourg, 2013

vanLehn et al., 2016

Olson, 2017

Holstein, Alevan, McLaren, 2018



LiveLab

- Informs teachers in real time about which students are struggling
- Early identification of students who are unlikely to master
- Teachers take ownership of the software
 - Mark as helped

Piecewise Functions

MAATHIA piecewise_functions_linear_use System Help Glossary Curriculum browser

< Unit Overview **Step-by-Step** Sample Problem Hints Solve It Progress I'm Done

You purchase your first cell phone. You are very excited because the phone case is covered with sparkly bling. Your parents agreed to pay for the monthly service plan and your data plan. You have to earn the money to pay for texting each month.

- The first 50 texts each month are free.
- Any texts over 50 cost \$0.02 each.

Let t represent the number of texts sent per month.
Let $c(t)$ represent the monthly cost for texts.

Define each piece of the function. Then, graph the function.

$$c(t) = \begin{cases} \text{[]} & \text{if []} \\ \text{[]} & \text{if []} \end{cases}$$

Use the piecewise function to answer each question.

1. How much will it cost to send 42 texts in a month?
Select the statement that best describes the solution.
2. If you paid \$1.7 in one month for your texts, how many texts did you send?
Select the statement that best describes the solution.

Monthly Cost dollars

Problem: pRad-071 Client version: 3.1.40 Server version: 3.1.40 © 2018 Carnegie Learning



Piecewise Functions

CL MATHia[®] piecewise_functions_linear_use System Help Glossary Curriculum Browser

< Unit Overview Step-by-Step Sample Problem Hints Solve It Progress I'm Done

You purchase your first cell phone. You are very excited because the phone case is covered with sparkly bling. Your parents agreed to pay for the monthly service plan and your data plan. You have to earn the money to pay for texting each month.

- The first 50 texts each month are free.
- Any texts over 50 cost \$0.02 each.

Let t represent the number of texts sent per month.
Let $c(t)$ represent the monthly cost for texts.

$$c(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq 50 \\ 0.02(t-50) & \text{if } t > 50 \end{cases}$$

Graph

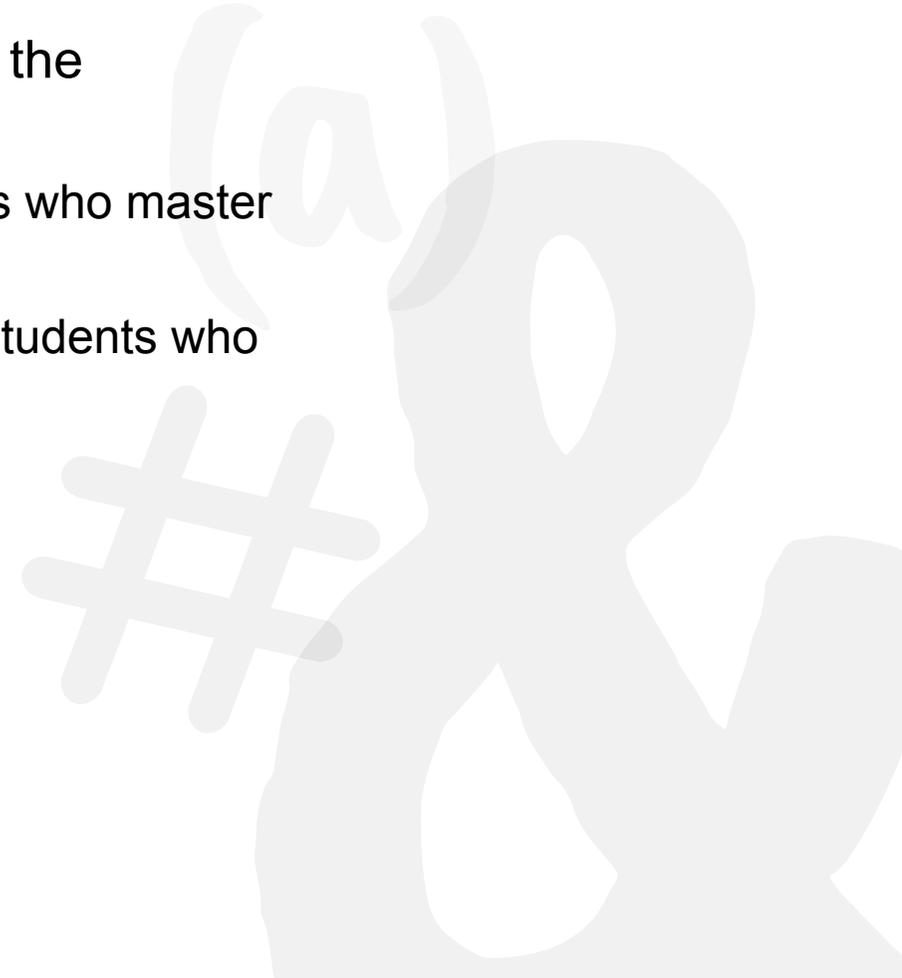
Use the piecewise function to answer each question.

1. How much will it cost to send 42 texts in a month?
Select the statement that best describes the solution.
The solution is a single value.
Select the piece of the piecewise function that is used to determine the solution.
 $c = 0$ if $0 \leq t \leq 50$
Enter the cost to send 42 texts in a month.
0 dollar(s)
2. If you paid \$1.7 in one month for your texts, how many texts did you send?
Select the statement that best describes the solution.
The solution is a single value.
Select the piece of the piecewise function that is used to determine the solution.
 $c = 0.02(t-50)$ if $t > 50$

- 2892 students
- 82.3% master
- Problem variety
 - 2 or 3 pieces
 - Function can be
 - $f(x)=c$
 - $f(x) = ax$
 - $f(x) = a(x-b)$

Criticality

- The extent to the knowledge component is the determining factor in mastery
 - For students who master: percent of students who master the KC in the last problem
 - For students who do not master: percent of students who do not master the KC



Most Critical KC

MATHia® piecewise_functions_linear_use System help Glossary Curriculum browser
Unit Overview Step-by-Step Sample Problem **Units** Solve It Progress I'm Done

You purchase your first cell phone. You are very excited because the phone case is covered with sparkly bling. Your parents agreed to pay for the monthly service plan and your data plan. You have to earn the money to pay for texting each month.

- The first 50 texts each month are free.
- Any texts over 50 cost \$0.02 each.

Let t represent the number of texts sent each month.

Let c represent the monthly cost.

Skills Progress to Mastery

Skills	Progress to Mastery
Define domain of piece.	<input type="text"/>
Define piece using constant expression.	<input type="text"/>
Define piece using simple variable expression.	<input type="text"/>
Define piece using more complex variable expression.	<input type="text"/>
Determine nature of independent quantity solution.	<input type="text"/>
Determine nature of dependent quantity solution.	<input type="text"/>
Select function piece relevant to independent quantity solution.	<input type="text"/>
Select function piece relevant to dependent quantity solution.	<input type="text"/>
Enter independent quantity solution.	<input type="text"/>
Enter dependent quantity solution.	<input type="text"/>

Monthly Cost

Number of Texts per Month

$$c(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq 50 \\ 0.02(t - 50) & \text{if } t > 50 \end{cases}$$

Graph

Use the piecewise function to answer each question.

1. How much will it cost to send 42 texts in a month?

Select the statement that best describes the solution.

The solution is a single value.

Select the piece of the piecewise function that is used to determine the solution.

$c = 0$ if $0 \leq t \leq 50$

Enter the cost to send 42 texts in a month.

0 dollar(s)

2. If you paid \$1.7 in one month for your texts, how many texts did you send?

Select the statement that best describes the solution.

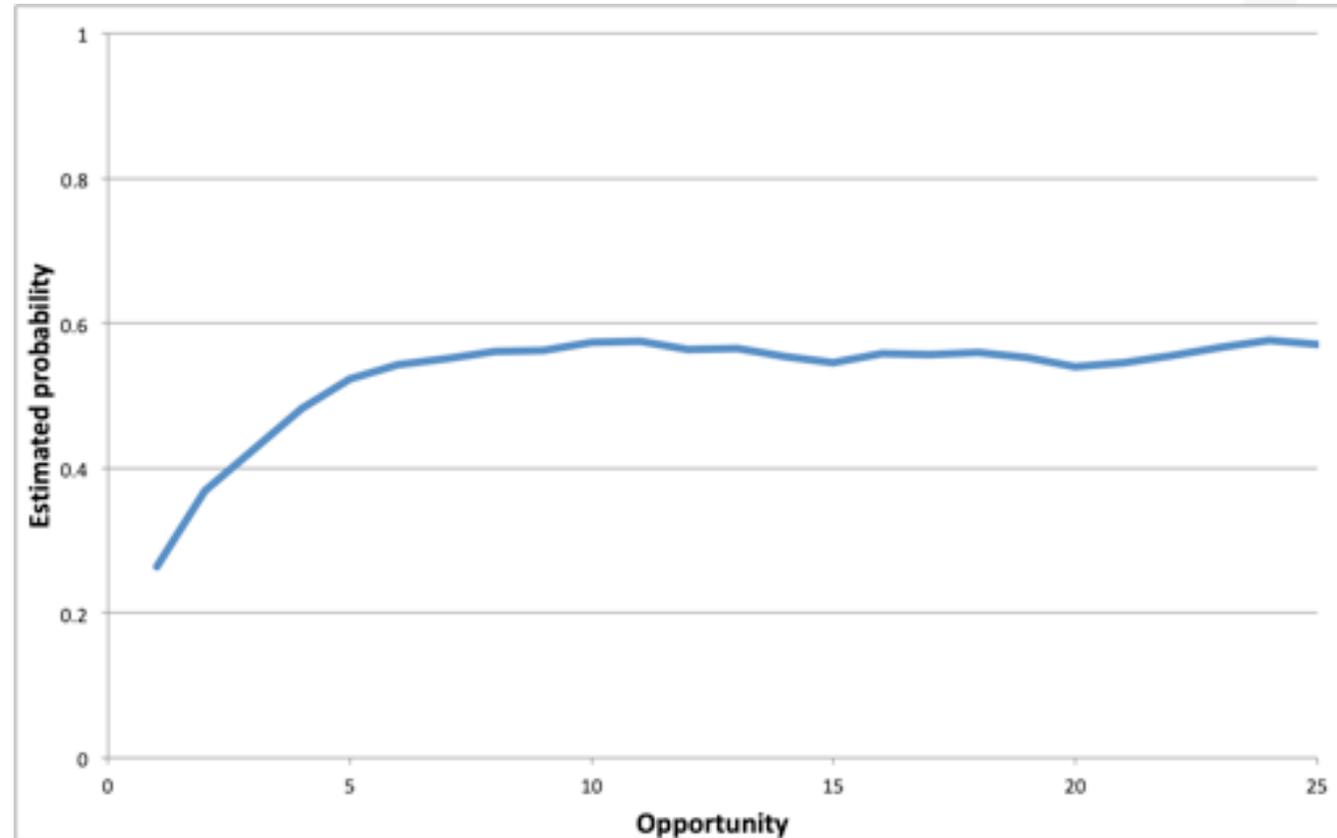
The solution is a single value.

Select the piece of the piecewise function that is used to determine the solution.

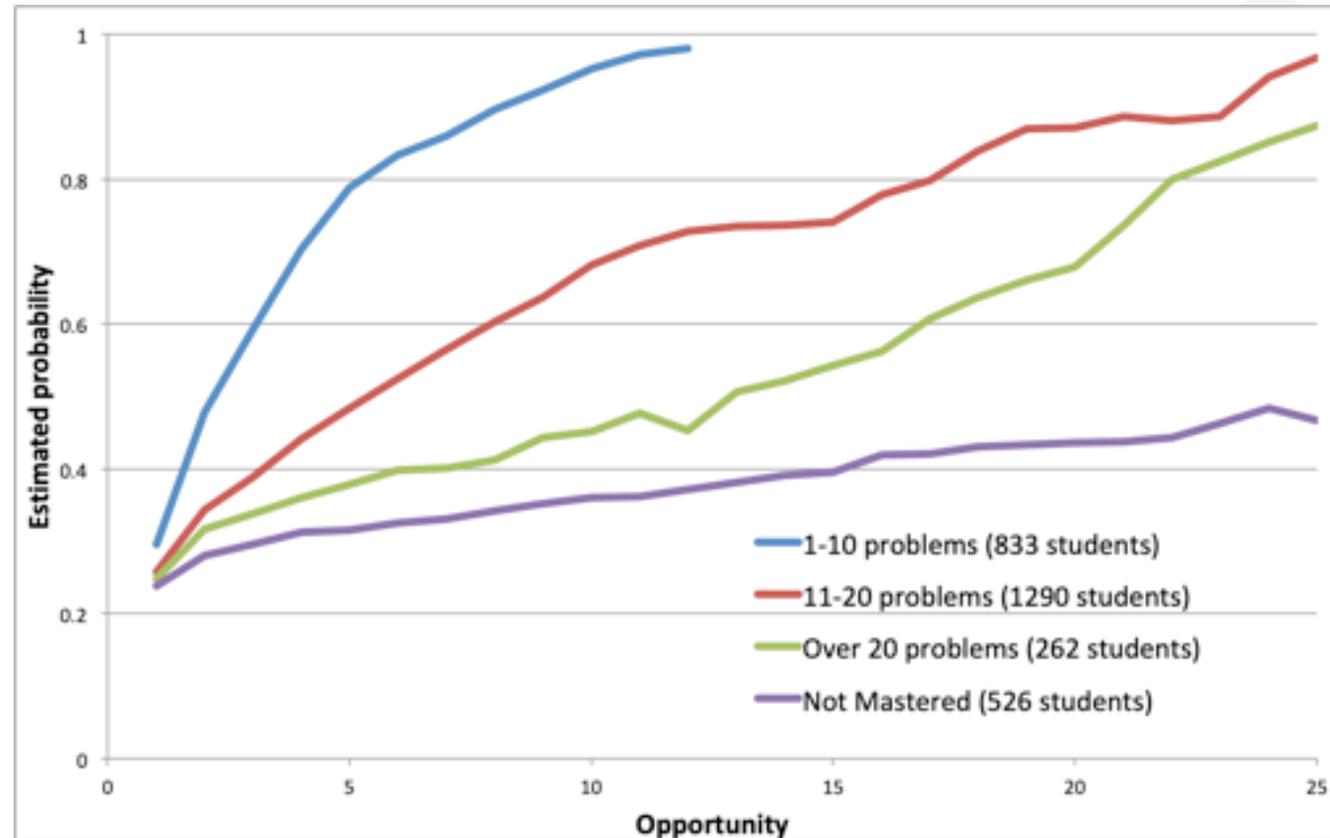
$c = 0.02(t - 50)$ if $t > 50$

System: p02-013 Client version: 3.3.40 Server version: 3.3.40 © 2018 Carnegie Learning

Learning curve – critical KC



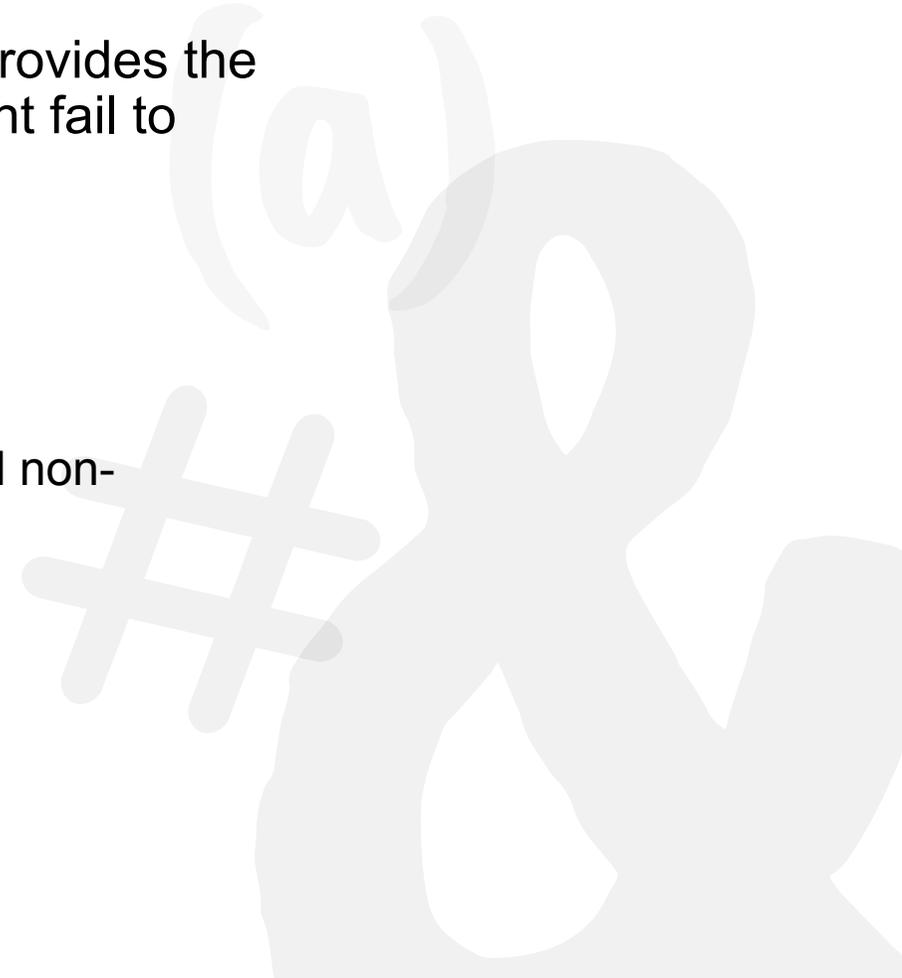
Segmented Learning Curve – critical KC



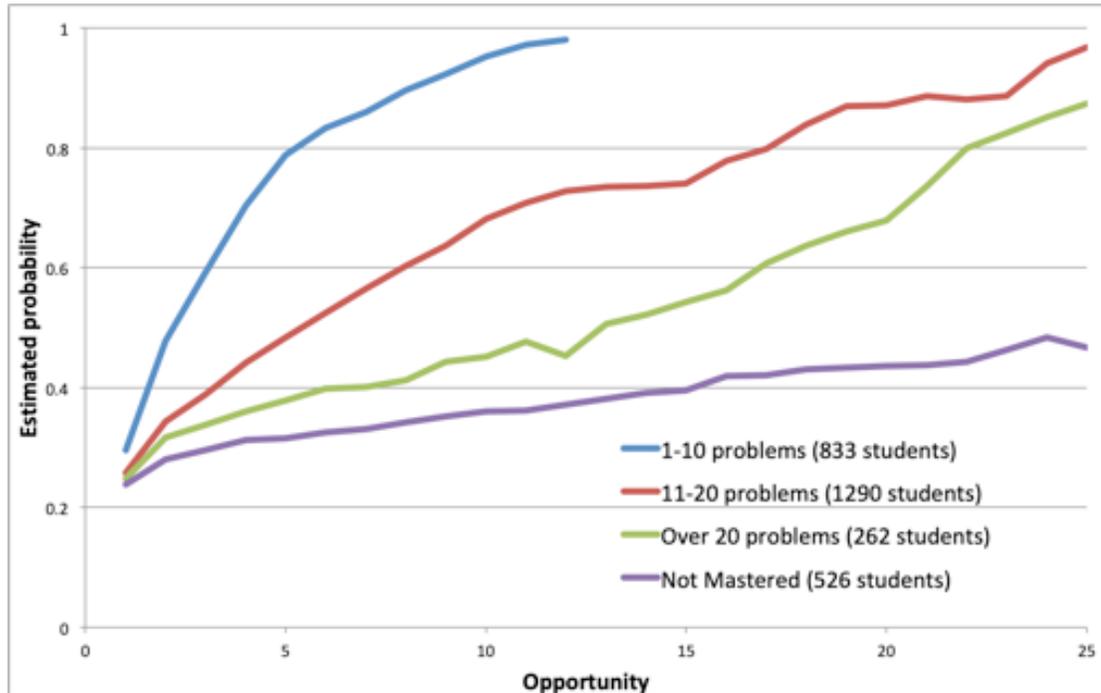
Murray et al., 2013

Indicator

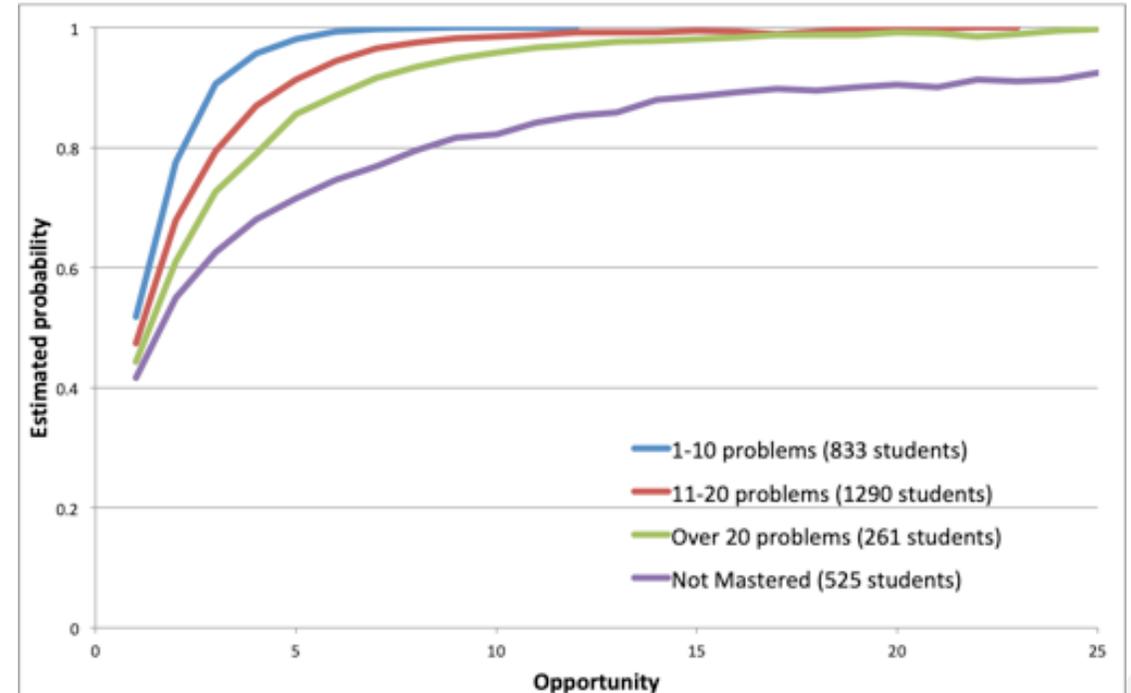
- Indicator knowledge component: KC that provides the earliest reliable warning that a student might fail to master
- Best indicators
 - Strongly discriminate between mastering and non-mastering students
 - Discriminate early



Critical and indicator KCs



- Critical for 67.2% of students
- 12.7% of students who fail to master workspace master this KC



- Critical for 18.1% of students
- 91% of students who fail to master workspace master this KC

Indicator KC

MATHia™ piecewise_functions_linear_use

System Help Glossary Curriculum Browser

< Unit Overview Step-by-Step Sample Problem Hints Solve It Progress I'm Done

You purchase a phone case for the month. Let t represent the number of texts you send in a month. Let $c(t)$ represent the monthly cost for texts.

Determine nature of independent quantity solution.

Determine nature of dependent quantity solution.

Select function piece relevant to independent quantity solution.

Select function piece relevant to dependent quantity solution.

Enter independent quantity solution.

Enter dependent quantity solution.

Define each piece of the function. Then, graph the function.

$$c(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq 50 \\ 0.02(t - 50) & \text{if } t > 50 \end{cases}$$

Graph

Use the piecewise function to answer each question.

1. How much will it cost to send 42 texts in a month?

Select the statement that best describes the solution.

The solution is a single value.

Select the piece of the piecewise function that is used to determine the solution.

texts in a month.

$c = 0$ if $0 \leq t \leq 50$

$c = 0.02(t - 50)$ if $t > 50$

2. If you have \$3.00 for your texts, how many texts did you send?

Select the statement that best describes the solution.

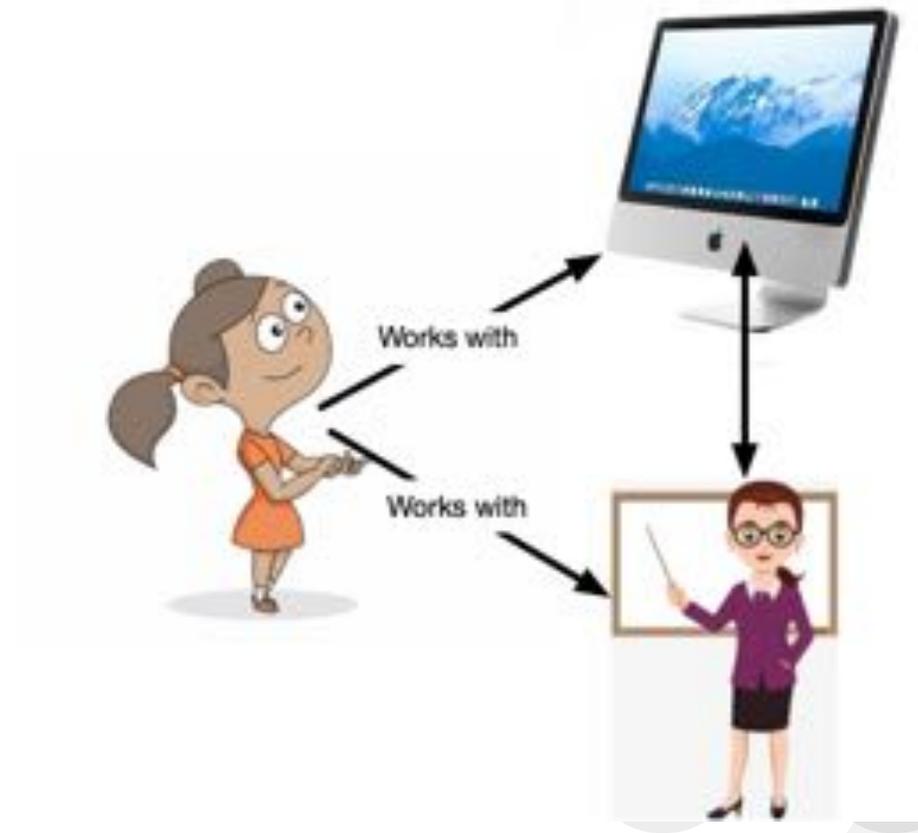
Monthly Cost dollars

Number of Texts per Month

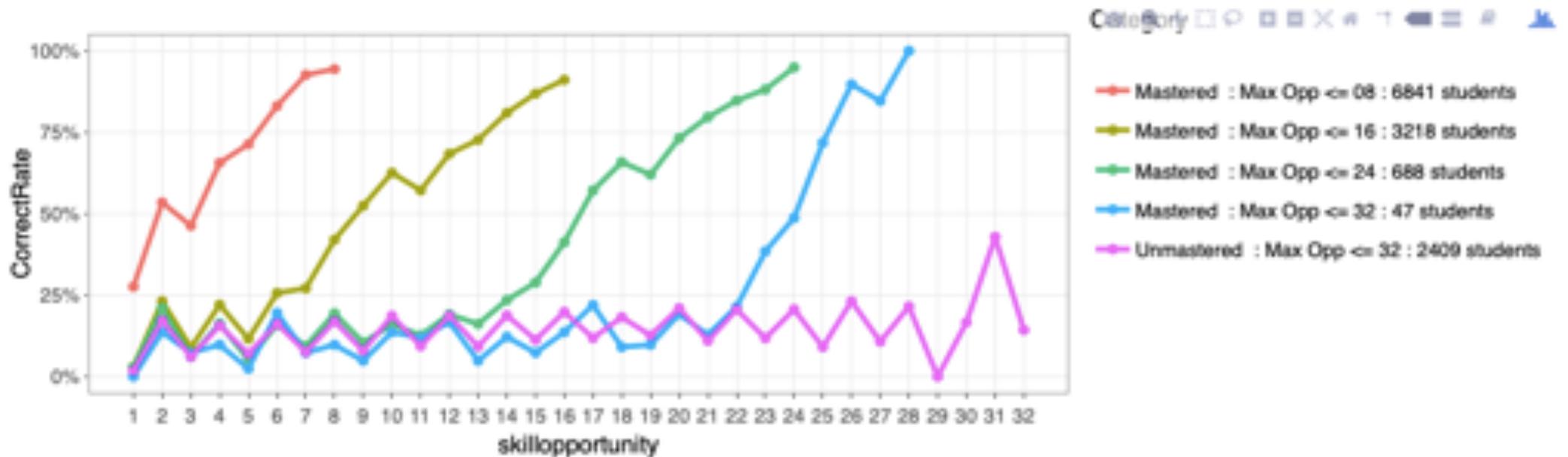
Problem: pfw2-073 Client Version: 3.3.40 Server Version: 3.3.40

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Completing the system



Different models for different students



EDM supporting favorable learning conditions

Design Adaptations

- Focus: Improvements that affect the largest number of students
- Timeframe: long

Learning Analytics

- Focus: Optimizing planning time, institutional decisions
- Timeframe: Medium

Teacher-in-the-loop orchestration

- Focus: Optimizing in-class teacher time
- Timeframe: short

Conclusion

- What makes a good teacher-in-the-loop intervention?
 - Applies to a subset of students
 - Represents a fundamental misconception or knowledge gap that teacher can address
 - Can be easily understood by teacher
 - And informs software
 - Can apply infrequently
 - Don't overload the teacher

