Using Inverse Planning for Personalized Feedback

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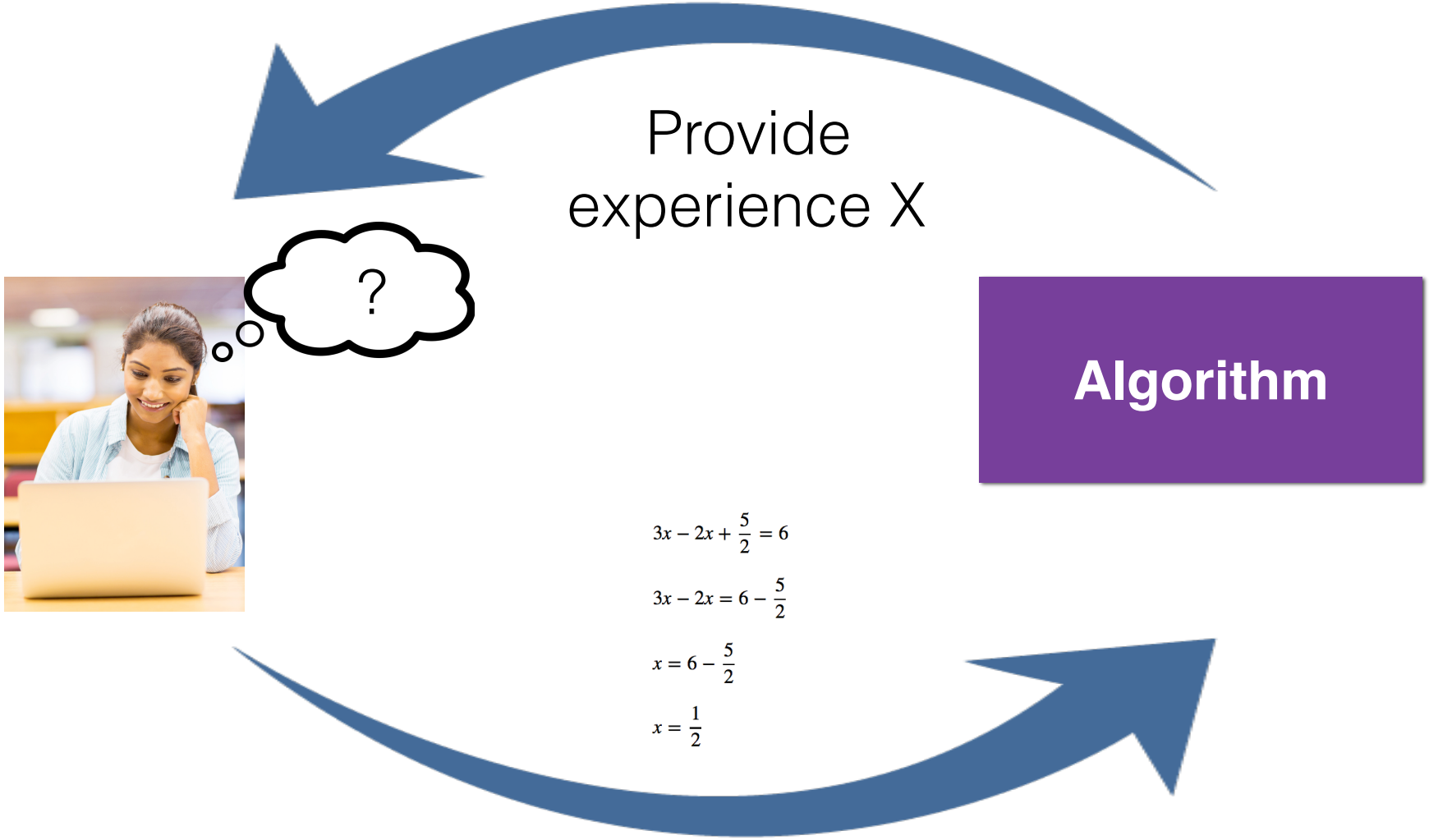
Using Data for Personalization



Outline

- Inverse planning: Diagnosing misunderstandings about equation solving
- Developing personalized feedback based on diagnosis
- Testing effectiveness of personalized feedback
- Future directions

Interpreting Equation Solving: Bayesian Inverse Planning

Algebra skills (θ_1)

$$3x + \frac{5}{2} = 2(x+3)$$

$$3x + \frac{5}{2} = 2x + 6$$

$$3x - 2x + \frac{5}{2} = 6$$

$$3x - 2x = 6 - \frac{5}{2}$$

$$x = 6 - \frac{5}{2}$$

$$x = \frac{1}{2}$$

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Algebra skills (θ_2)

$$\Theta = \text{space of possible understandings}$$

$$p(\theta \mid \text{equations})$$

Representing Understanding: Θ

$\theta \in \Theta$: 6-dimensional vector of parameters related to skill

Conceptual Mal-rules	1+3x => 4x 3(2+5x) => 6+5x
Arithmetic	1+5.9x+3.2x => 1+8.1x -3+5+x => -2+x
Planning	3x+5x+4 = 2 => 3x+4 = -5x+2

e.g., Sleeman, 1984; Payne & Squibb, 1990; Koedinger & MacLaren,1997

Bayesian Inverse Planning

Algebra skills (θ_1)

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Algebra skills (θ_2)

Bayesian Inverse Planning

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Algebra skills (θ_2)

$$p(\theta \mid \text{equations}) \propto \underbrace{p(\theta)}_{\text{Prior}} \underbrace{p(\text{equations} \mid \theta)}_{\text{Likelihood}}$$

Prior: Encode information about what misunderstandings are common

Bayesian Inverse Planning

Algebra skills (θ_1)

$$3x + \frac{5}{2} = 2(x+3)$$

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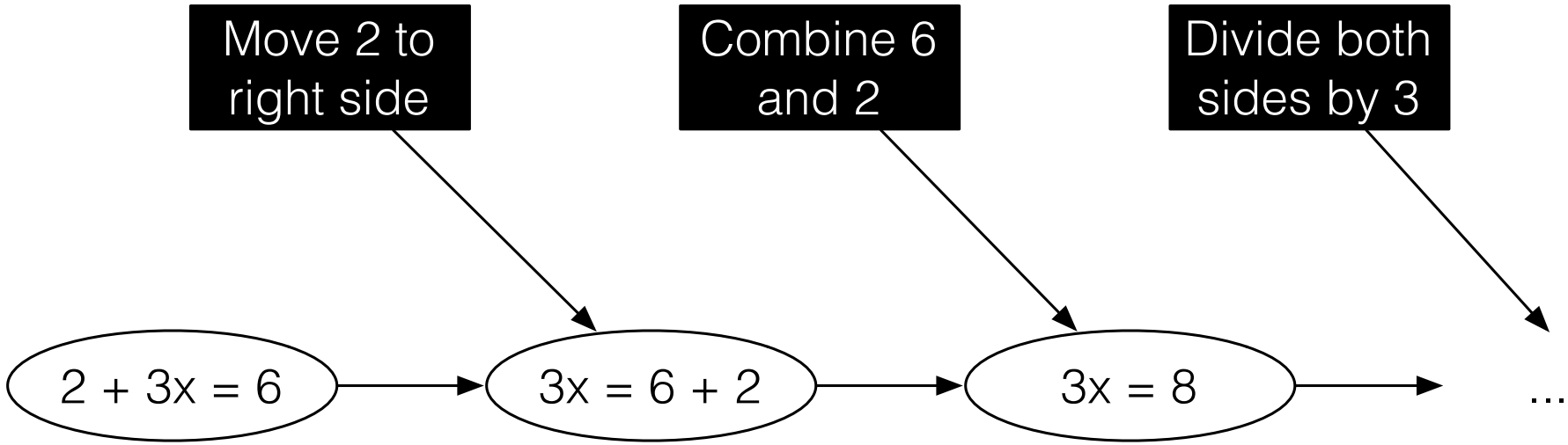
Algebra skills (θ_2)

$$p(\theta \mid \text{equations}) \propto \underbrace{p(\theta)}_{\text{Prior}} \underbrace{p(\text{equations} \mid \theta)}_{\text{Likelihood}}$$

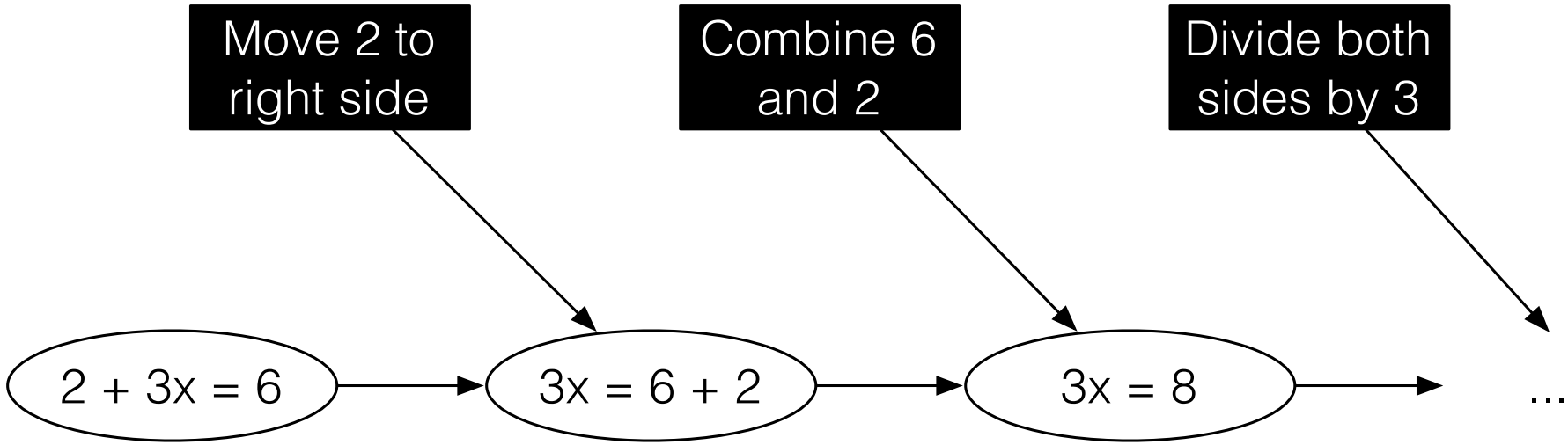
Likelihood: What is the probability of the observed data if the learner has a particular understanding?

Generative Model of Equation Solving: Markov Decision Processes



θ affects **what actions** are considered and **transition probabilities** for actions.

How are Actions Chosen?



Assume a noisily optimal policy:

$$p(a \mid s) \propto \exp(\theta_\beta \cdot Q(s, a))$$

Long term expected value:

$$Q(s,a) = \sum_{s' \in S} p(s'|s,a) \left(R(s,a) + \gamma \sum_{a' \in A} p(a'|s') Q(s',a') \right)$$

Inverse Planning Overview

Arithmetic error parameter

Distributive property error parameter

Move term error parameter

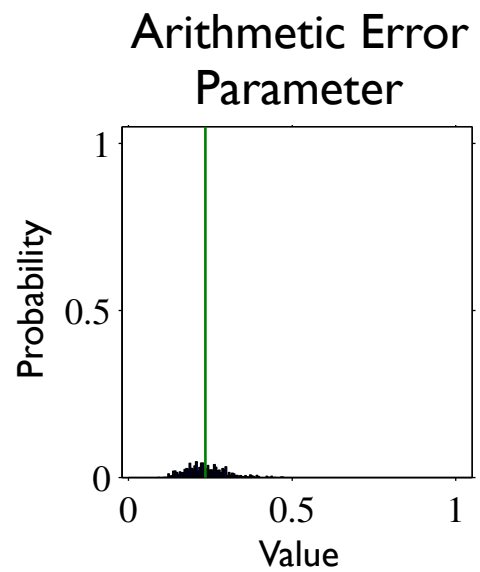
Action planning parameter

⋮

Representation of understanding (Θ)

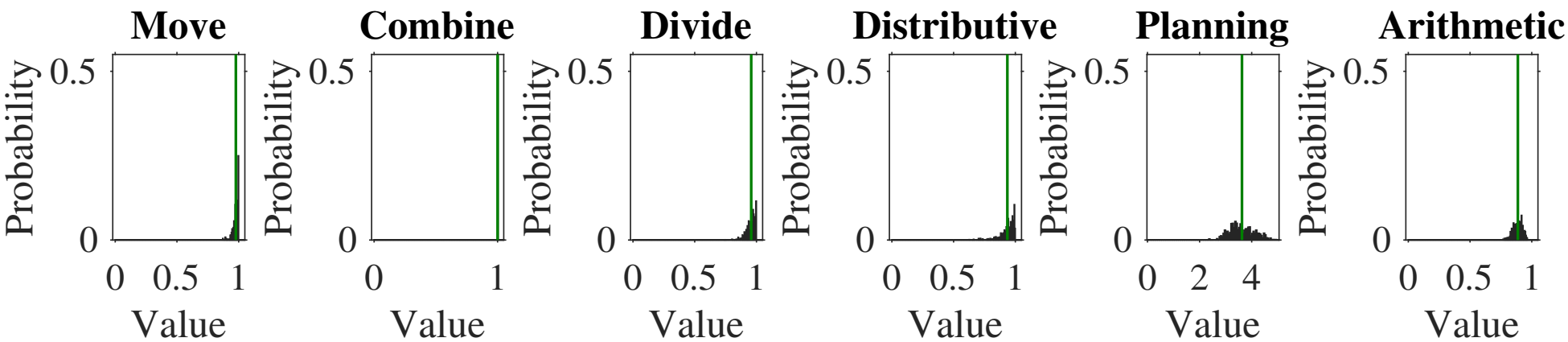
$$
\begin{aligned}
5+9 &= 6.0x + 2.0x + 10.0[1+1+7.0x] \\
5+9 &= 6.0x + 2.0x + 10 + 10 + 70.0x \\
5+9 &= 6.0x + 2.0x + 20 + 70.0x \\
14 &= 6.0x + 2.0x + 20 + 70.0x \\
5+9 &= 6.0x + 2.0x + 10.0[1+1+7.0x] \\
5+9 &= 6.0x + 2.0x + 10 + 10 + 70.0x \\
5+9 &= 6.0x + 2.0x + 20 + 70.0x \\
14 &= 6.0x + 2.0x + 20 + 70.0x \\
14 &= 76.0x + 2.0x + 20.0 \\
14 &= 78.0x + 20.0 \\
14 + -20 &= 78.0x \\
-7 &= 78.0x \\
-\frac{7}{78} &= 1x
\end{aligned}
$$

Model of equation solving as a (parameterized) MDP



Infer posterior probability over Θ (MCMC)

Output for One Learner



How do we turn this into a feedback activity?

Feedback Activities

You completed enough problems to get feedback from the tutor! The more problems you do, the more you'll improve your algebra skills. The computer analyzed the way that you have been solving problems to identify skills that you're close to mastering and skills where you still need some improvement.

Distributive property

Great! You're able to turn terms like *3(x + 4)* into *3x + 12*, distributing the *3* over the terms in parentheses. You know that you have to multiply *all* of the terms in parentheses by the coefficient.

Moving terms

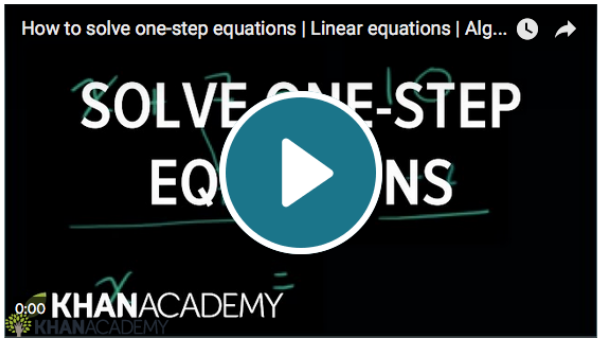
You sometimes have difficulty moving a term from one side of the equation to the other. When you move a term, you're subtracting that term from both sides. This means that the term will *change* signs when it moves from one side to the other.

You will now be reviewing a skill that you haven't quite mastered. Click continue to begin!

Overview of skills and assessment

To manipulate an algebra equation, you always have to do the same operations to both sides. Otherwise, the two sides will no longer be equal! When you move a term from one side of the equation to another, you're adding or subtracting it from each side. For instance, to move the *-2* in *-2+x=5* from the left side to the right side, you would add *2* to the left hand side, since *-2+2=0*, and you would add *2* to the right side. That makes the right side *5+2*. *The 2 ended up on the right side, with the opposite sign that it had on the left side. Watch the video below to learn more.*

Adding and subtracting the same thing from both sides: Two worked examples of solving simple equations



Text explanation and video from Khan Academy

most points for getting the problem right, but you still earn some points for trying! Remember, you can click the "interface reference" tab to get a remind... u're ready to continue!

Show interface help

It looks like you made an error. The first row where you have an error is highlighted. Check your work, making sure you always keep the two sides equal to one another. You might want to delete all of the rows below the one with the error, so that you won't be confused by your previous attempt. Remember, the more you practice your skills, the better you'll perform!

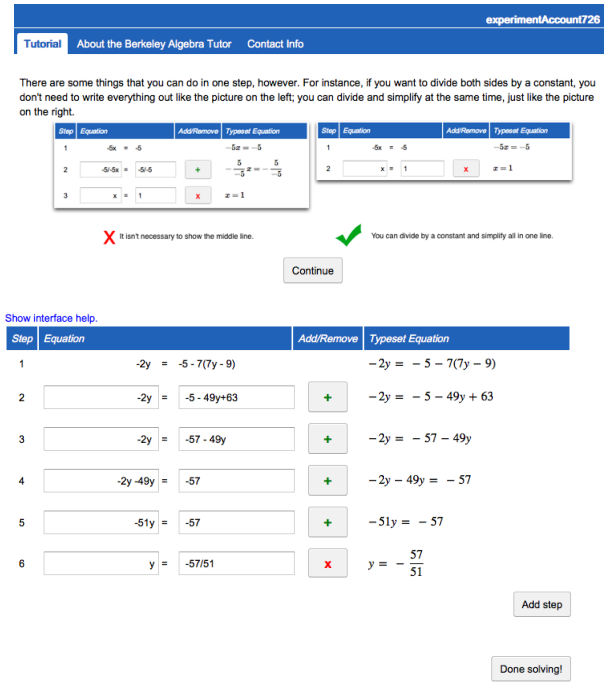
Try again

Step	Equation	
1	-4	20
2		4
3		
4	x = -24/5	x $x = -\frac{24}{5}$

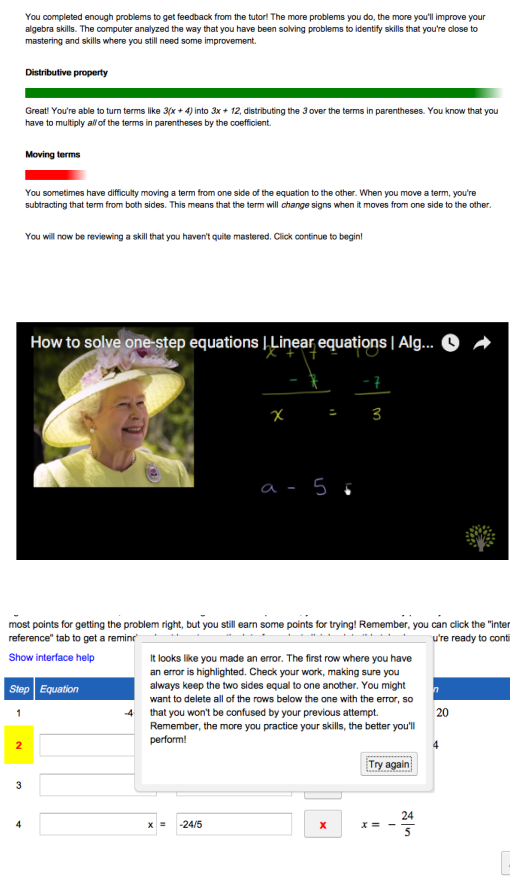
Add step

Targeted practice with fading scaffolding

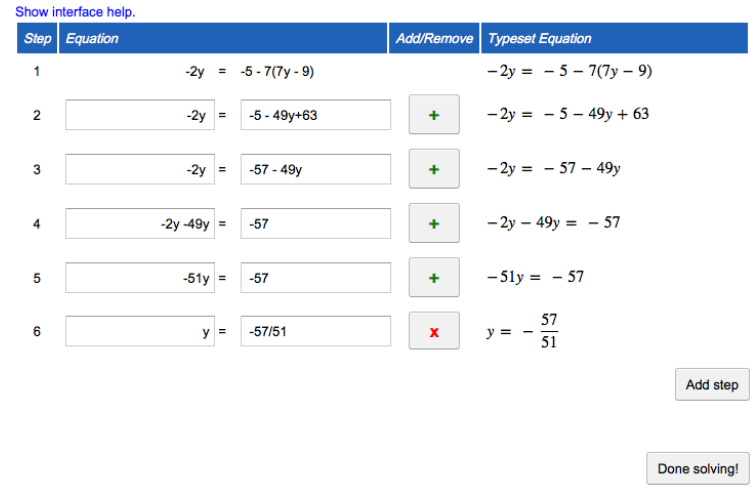
Testing Personalized Feedback



Session 1:
Website Problem Solving and Multiple Choice Test

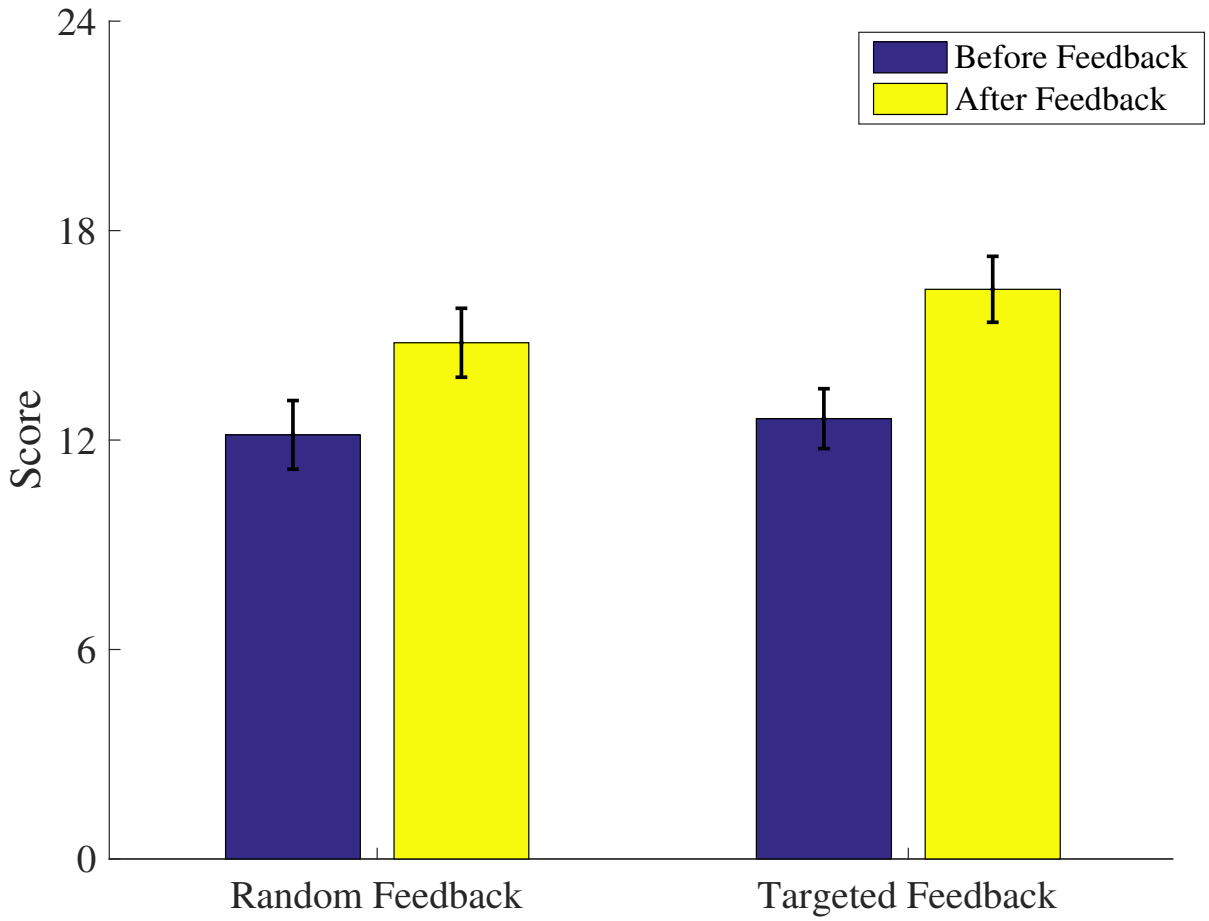


Session 2:
Feedback Activity



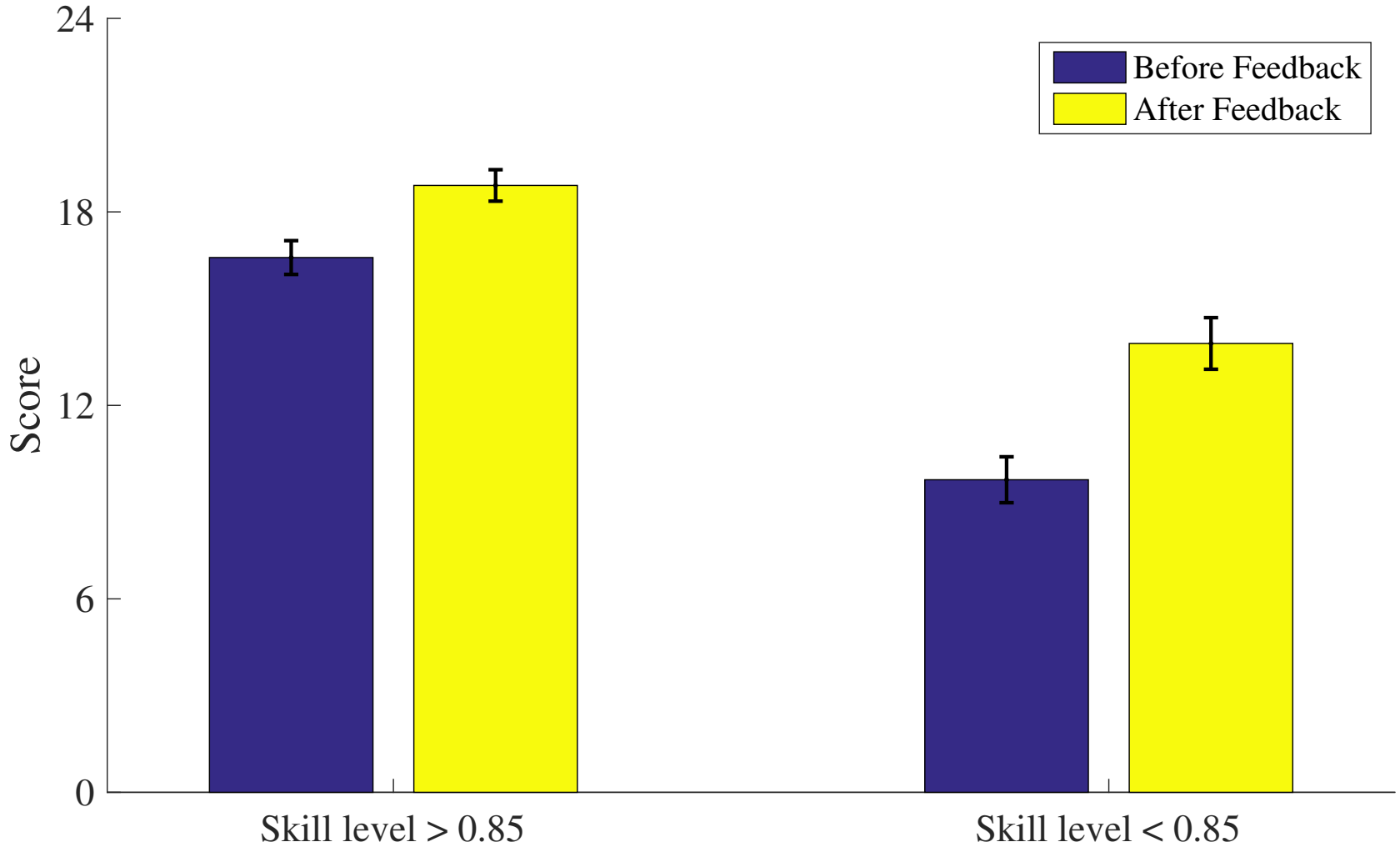
Session 3:
Website Problem Solving and Multiple Choice Test

Results:
Changes in Performance Across Sessions

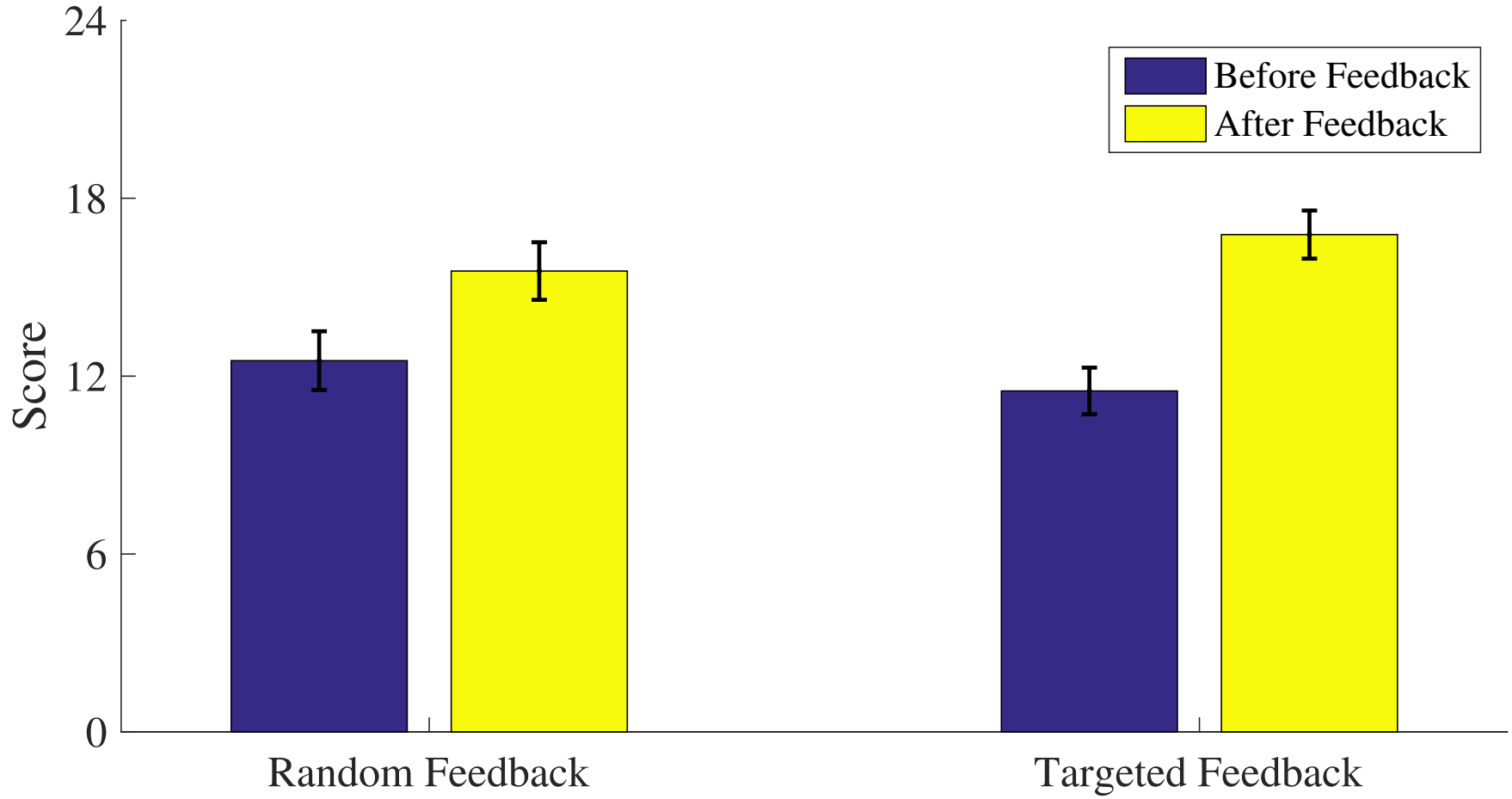


Reliable improvement, but no difference in amount of improvement across conditions.

Performance Based on Proficiency Level of Feedback Skill



Performance Change for Participants with Varying Skill Levels



Reliable difference in amount of improvement by condition.

Contributions and Next Steps

- Personalization using inverse planning is helpful for learners who struggle with only some skills
- Provides an applied metric assessing the algorithm
- Next steps:
  - Greater specificity and more interactivity in feedback
  - Longer term interventions

Thank you!

Contact: Anna Rafferty, arafferty@carleton.edu

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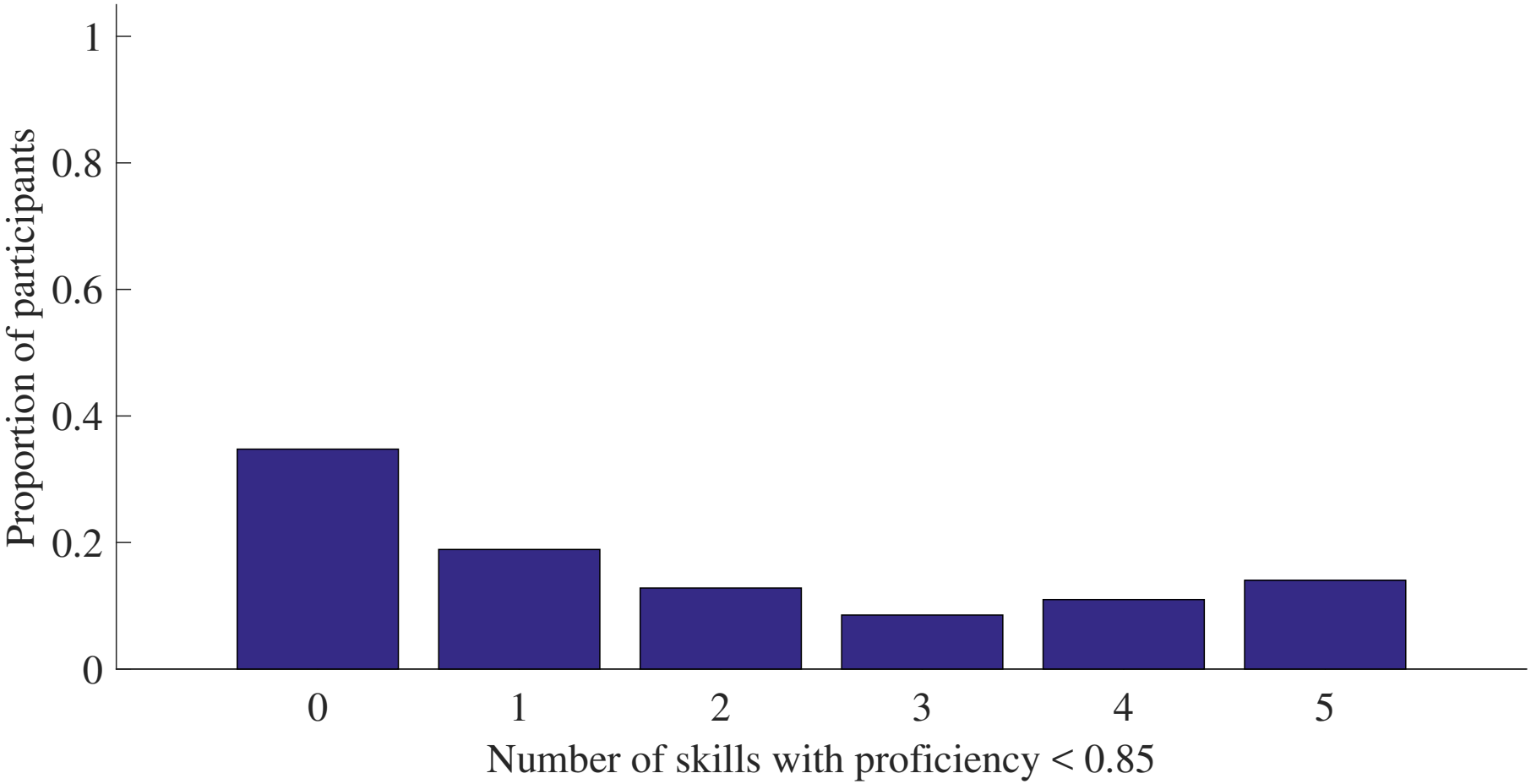
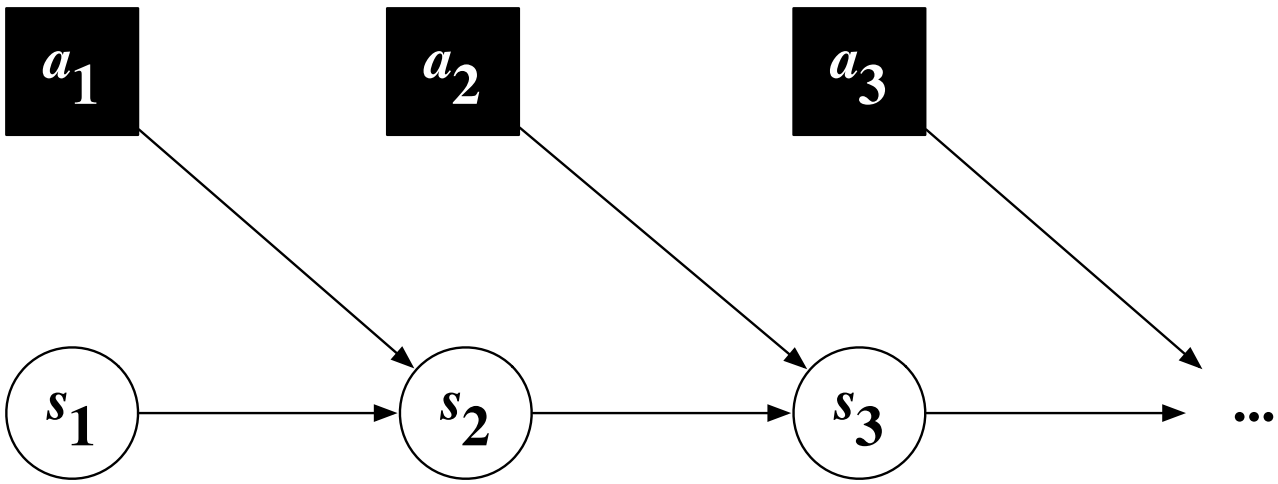
Skill Proficiencies by Participant



Markov Decision Processes



Actions:

- **move** a term
- **multiply or divide** by a constant
- **combine** two terms
- **distribute** a coefficient
- **stop** solving