Extracting Latent Skills from Time Series of Asynchronous and Incomplete Examinations

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ABSTRACT

Examinations are tools for measuring examinees' skills. A question item in an examination requires several skills to solve it. In order to grasp latent skills, it is important to find which skills an item requires. The relationship between items and skills can be represented by a Q-matrix. Recent studies have attempted to extract a Q-matrix by non-negative matrix factorization (NMF) from a set of examinees' test scores. In order to apply NMF, examination results without missing values are required as the matrix to be decomposed. However, it is difficult to assemble complete examination results because users of intelligent tutoring systems solve different items at different times. In this paper, we propose a method which extracts a Q-matrix by aggregating incomplete examination results asynchronously.

1. INTRODUCTION

The concept of a Q-matrix was developed on the basis of the rule space method (RSM) by Tatsuoka et al. [4]. A Q-matrix allows us to determine which skills are necessary to solve each item of an examination. Recently, there have been several studies on how to extract a Q-matrix from a set of examination results [1, 2]. These studies applied the non-negative matrix factorization (NMF) method to decompose the results of an examination into a Q-matrix and an S-matrix (which gives the relationship between skills and users). In particular, the online NMF with regularization (online NMF) has been proposed in order to extract a constant Q-matrix from examination time series in an online fashion [2]. Although these NMF methods require the input matrix to have no missing values in order to be able to factorize it, real examination results have many missing values because users of general intelligent tutoring systems (ITSs) do not always solve all items. In this paper, we introduce a novel method for extracting a time-invariant Q-matrix from a time series of asynchronous and incomplete examination results. The key ideas are as follows: 1) to synchronize time-series data per *"stage"* (the period during which a fixed number of items are given) to obtain multiple item-user matrices having missing values and 2) to apply the weighted NMF [3] to each matrix in an online manner as with the online NMF. We empirically demonstrate the effectiveness of the proposed method by using artificial data sets.

2. COLLECTING EXAMINATION RESULTS FROM REAL ITS

In the Q-matrix extraction by the online NMF [2], the Q-matrix $\mathbf{Q}$ was assumed to be constant, while S-matrix $\mathbf{S}_t$ varied over time because users acquired knowledge by learning and experiences. An examination result $\mathbf{R}_t$ changed whenever $\mathbf{S}_t$ changed because $\neg\mathbf{R}_t$ was obtained according to the equation $\neg\mathbf{R}_t = \mathbf{Q} \circ (\neg\mathbf{S}_t)$, where the operator $\neg$ denotes Boolean negation. In order to extract a constant Q-matrix from such variable examination results, the key idea of the online NMF was that the initial values of the matrix are inherited from the Q-matrix of the previous decomposition; this is in contrast to the conventional NMF-based method, in which the initial values are set at random.

However, in a real ITS, it is impossible to collect all users' answers as examination results because each user solves different items at different times, even the online NMF needs all answers. In this paper, we introduce the novel concept of a *stage* to resolve this problem. We define one stage as a period during which a fixed number of items are given and a fixed number of skills are required for each stage. In this paper, one stage is defined as when items in an ITS are given c items to a user. Elements of $\neg\mathbf{R}_s$ are collected as users' results from every stage s. The overall flow of the proposed method is shown in Figure 1. In this figure, the $\neg\mathbf{R}_s$ are constructed using a stage with $c = 3$.

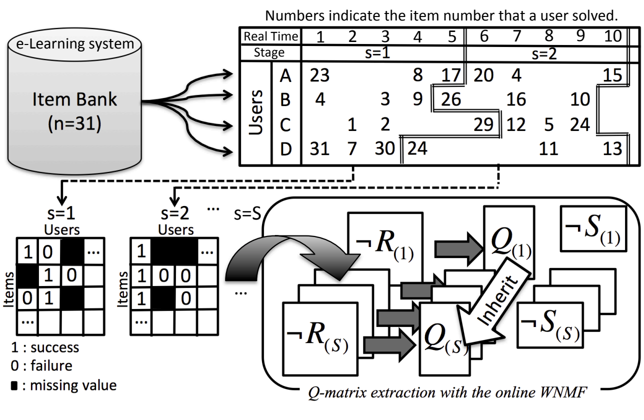

Figure 1: Overall flow of the proposed method

3. Q-MATRIX EXTRACTION FROM INCOMPLETE EXAMINATION RESULTS

Whereas the online NMF needs a filled matrix, the examination results collected as $\neg\mathbf{R}_s$ have missing values. In order to factorize the incomplete matrix, we apply the WNMF. The WNMF can cope with missing values in an observed matrix. Suppose $\mathbf{W}$ is a binary matrix of the same size as $\neg\mathbf{R}$ such that $\mathbf{W}_{ij} = 1$ when $\neg\mathbf{R}_{ij}$ is known and $\mathbf{W}_{ij} = 0$ when $\neg\mathbf{R}_{ij}$ is missed. The update rules of the extraction of the Q-matrix with WNMF are as follows:

$$\mathbf{Q}_{ik} \leftarrow \mathbf{Q}_{ik} \frac{((\mathbf{W} * \neg\mathbf{R})\neg\mathbf{S}^\top)_{ik}}{((\mathbf{W} * (\mathbf{Q}\neg\mathbf{S}))\neg\mathbf{S}^\top)_{ik}}, \quad (1)$$

$$\neg\mathbf{S}_{kj} \leftarrow \neg\mathbf{S}_{kj} \frac{(\mathbf{Q}^\top(\mathbf{W} * \neg\mathbf{R}))_{kj}}{(\mathbf{Q}^\top(\mathbf{W} * (\mathbf{Q}\neg\mathbf{S})))_{kj}}, \quad (2)$$

where $*$ denotes element-wise multiplication.

The online WNMF produces a Q-matrix by letting the initial values be those obtained at the previous stage. The cost function of the online WNMF can be written as

$$\min_{\mathbf{Q}_s, \mathbf{S}_s} \{\|\neg\mathbf{R}_s - \mathbf{Q}_s\neg\mathbf{S}_s\|_F^2 + \lambda(s)(\|\mathbf{Q}_{s-1} - \mathbf{Q}_s\|_F^2)\}, \quad (3)$$

where $\lambda(s)$ is a monotonic increasing function of time given by $\lambda(s) = \alpha s/S$, where s is a stage in $(1, \ldots, S)$, and α is the constant parameter determining the rate of increase. At each stage s, we find $\mathbf{Q}_s$ and $\neg\mathbf{S}_s$ according to (3), so that the sum of the factorization error and regularization term is minimized. In the optimizations with respect to $\mathbf{Q}_s$ and $\neg\mathbf{S}_s$, we set $\mathbf{Q}_s$ by inheriting $\mathbf{Q}_{s-1}$, and choose $\neg\mathbf{S}_s$ by taking random non-negative values.

4. EXPERIMENTAL RESULTS

In order to verify the effectiveness of our methods, we made a synthetic examination time series. We generated a time-varying S-matrix and a fixed Q-matrix to obtain $\neg\mathbf{R}_s$ according to the equation $\neg\mathbf{R}_s = \mathbf{Q} \circ (\neg\mathbf{S}_s)$. A conjunctive Q-matrix consisted of 31 items and 5 skills. We designed a time series of $\neg\mathbf{S}_s$ as a process of acquiring skills, on the basis of the item response theory.

As a measure of the performance for Q-matrix extraction, we introduce a Q-matrix error e_s between $\mathbf{Q}$ and an extracted matrix $\widehat{\mathbf{Q}}_s$ as $e_s = \|\widehat{\mathbf{Q}}_s - \mathbf{Q}\|_F^2$. Note that the factorized solutions obtained using the NMF may not be unique due to the randomness of the initial matrices. Hence, we calculated the mean and standard deviation of Q-matrix errors from 10 simulations.

To begin with, we need to investigate the factorized performance of WNMF which concerns with the missing rate of an input matrix. The matrix was made by simulating random $\{0, 1\}$ as a filled matrix. We made incomplete matrices from the matrix by giving various masks with different missing rate to it. Figure 2 shows the relationship between factorized errors and missing rates of matrices. The factorized errors are low when the missing rates of matrices are less than 30%. As a result, we defined one stage as $c = 23$, namely the missing rate of each $\mathbf{R}_s$ was 25% because a user solved 23 out of 31 items. Figure 3 shows the Q-matrix errors for both the WNMF and the online WNMF for examination results having a missing rate of 25%. Although,

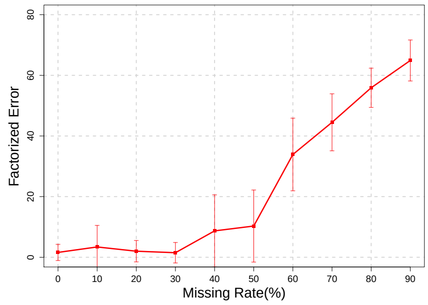

Figure 2: The correlation between factorized errors and missing rates of an input matrix.

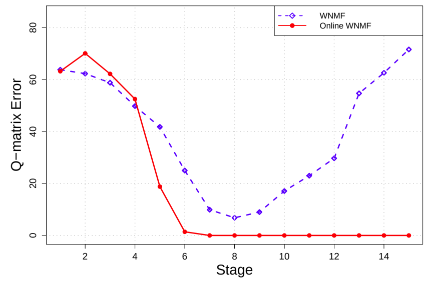

Figure 3: The Q-matrix errors with the WNMF and the online WNMF over missing rate 25%.

in the WNMF only, the Q-matrix error did not become zero at any stage and the error gradually increased after stage=8, the online WNMF overcame this problem.

5. CONCLUSIONS

In this paper, we have introduced the concept of a stage to collect users' answers asynchronously and have proposed the online WNMF for the purpose of extracting a constant Q-matrix from a time series of incomplete examination results. We have designed the method so as to decompose examination results with missing values. Finally, we applied the proposed method to a synthetic data set to demonstrate that it could find a constant Q-matrix stably.

6. ACKNOWLEDGMENTS

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